

Can Endogenous Participation Explain Price Volatility? Evidence from an Agent-Based Cobweb Model

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Abstract This work provides a computational cobweb model with heterogeneous adaptive producers with endogenous market entry and exit. Firms face a borrowing constraint and so can go bankrupt. At the same time when average profits are positive there is an inflow of new firms in the market. Bounded dynamics and endogenous volatility are shown to follow without resorting to nonlinearities.

Keywords Heterogeneous agents · Expectations · Price instability · Market entry and exit

1 Introduction

The dynamic evolution of commodity prices and in particular their volatility carries several important consequences in the lives of people and therefore understanding such phenomena is both interesting and urgent from the standpoint of the economics profession. One important class of models that have been used to this end is that of the cobweb models, which posit that suppliers have to make a decision about production significantly before its actual realization. This results in a short-run inelastic supply. For example some types of agricultural markets are characterized by biological lags that naturally suggest the use of cobweb models. The first studies of the cobweb model under bounded rationality go back to Ezekiel (1938), Nerlove (1958) and Carlson (1968) several decades ago. These studies introduced the linear cobweb model under various restrictive assumptions, e.g. about expectations, which were naive or adaptive, and the supply side, which was modelled through a representative firm. Various

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criticism has been raised towards these models. A comprehensive survey is contained in Gouel (2010). One issue is that the kind of dynamic patterns that can be generated, namely convergence, explosive paths and two-cycles are rather poor with respect to the observed time series of prices. Introducing nonlinearities in the supply and/or the demand and more elaborate assumptions on expectations helps indeed overcoming such difficulties. Brock and Hommes (1997) introduced the possibility for firms in the cobweb to choose between costly rational expectations and naive expectations, with a stochastic discrete choice model governing the possibility of switching between the two. They found evidence of complex dynamics within an adaptively rational equilibrium (the so-called rational route to randomness). Later, Goeree and Hommes (2000) generalized this model to nonlinear (monotonic) demand and supply, while Branch (2002) introduced a kind of adaptive expectations (or rather a fading memory mechanism) within the menu of available predictors. On a slightly different position, Colucci and Valori (2011) have shown how introducing heterogeneous adaptive expectations in a simple cobweb can trigger unexpected dynamic consequences with respect to the correspondent homogeneous model.

Another weakness characterizing many cobweb models, both linear and non-linear, has to do with the fact that the financial side of the firm's behavior receives a somewhat careless treatment, often merely (and implicitly) consisting in the possibility of accessing unlimited borrowing to rule out bankruptcy. This point has been raised by Commendatore and Currie (2008), who have developed a fully fledged model of the financial sector responsible for providing liquidity for the firms. They have shown a connection between unconstrained borrowing and financial crises and suggested that having some constraint on borrowing is required to make such models credible. An earlier cobweb model in which firms have limited capacity of varying output at will as a result e.g. of financial constraints can be found in Huang (1995).

A further element which deserves attention is the fact that there are very few attempts to model endogenous participation in the cobweb literature. An exception is given by Dawid and Kopel (1998), who used genetic algorithms as a way of representing firms' choices among which the decision to stay in the market or leave. In this paper we aim at treating the issue of participation in the market as an endogenous output of the model rather than as a given. On one hand this implies taking the issue of firms' financial viability seriously, having bankrupt firms eventually leave the market. On the other hand there is the issue of modelling market entry decisions. It would appear reasonable to assume that entrepreneurial decisions to entry should be strongly connected to the market's capacity to host new players and to the level of existing competition. Such assumption however would be typically wrong in many empirical contexts. Empirical evidence shows for example that entry is common but the survival rate of most entrants is low, and even successful entrants may take quite a long time to achieve a size comparable to the average incumbent (see Geroski (1995)). Indeed a significant amount of overconfidence in market-entry decisions has been documented by Busenitz and Barney (1997) and a general tendency to neglect competition has been found more recently by Simonsohn (2010) for sellers in the online auction context. Anecdotal evidence, such as the abundance of internet startups right before the dot.com bubble was about to burst at the end of last century, also add to this picture. Moore et al. (2007) have shown that market-entry decisions are biased by the presence of a widespread

excessive self-focus with respect to a careful assessment of the prospective market's environment and competition.

In the present paper we describe an agent-based computational cobweb model with borrowing constraints and endogenous participation: this means that the flow of firms' profit accumulate and define their financial wealth, a stock variable. If wealth becomes negative (due to periods of losses) a firm can access borrowing up to a limit. Past such threshold the firm goes bankrupt and exits. So our assumption is that as soon as a firm cannot repay its debts it is forced to leave the market. This is indeed a very simple way of modelling credit constraints, which allows nonetheless comparative statics analysis of the impact of variations of such threshold. A thorough treatment of bankruptcy and insolvency clearly goes beyond the scope of the present paper: it would require evaluating a firm's net assets, i.e. the possibility of selling illiquid assets to cover the liabilities. This would in turn pose the problem of (determining and knowing) the present and future prices of such illiquid assets (see Kiyotaki and Moore (1997) for a model of this kind).

At the same time at each period there is a pool of potential entrants, each of which has a constant positive probability of becoming a startup in the market, provided the mean profit of incumbent firms over the last τ periods is non-negative. When mean profits over the last τ periods are negative there is no entry. This is our way of making operational the notion that there is only a bland correlation between the amount of expected profits and the entry rate.

With respect to the literature assuming nonlinearities and heterogeneous firms switching between different predictors (e.g. Brock and Hommes 1997) our structure is simpler, given that the model's main message remains valid even with linear demand and supply and simple adaptive heterogeneous firms as in our previous paper (Colucci and Valori 2011). For what concerns the financial side, our model is similar to one of the scenarios studied by Commendatore and Currie (2008): the difference is that Commendatore and Currie (2008), whose modelisation of the banking sector is admittedly more sophisticated, impose a zero net entry rate (i.e. that firms leaving the market are substituted by an exact number of entrants) to ensure tractability, whereas we do not. As a result the model can be simulated to yield several interesting observations: in particular the mechanism of entry and exit of firms induces endogenous price volatility. Therefore, with respect to the underlying linear model without entry and exit an interesting form of economic dynamics is generated, in which the price evolution remains bounded but oscillates erratically and can display booms and busts.

2 The Model

The backbone of the model is quite standard. Consider a cobweb-type commodity market in which the demanded quantity is a function of the current price,

$$Q_t = D(p_t) = a - bp_t.$$

Each firm needs to allow for a production lag and so choose optimal supplied quantities conditioned on the forecasted future price $p_{i,t}^e$, which characterizes firm i belonging

to the market at time t . The set of firms active in the market at time t shall be denoted by M_t . The optimal supply solves the expected profit maximization problem

$$\max_{q_{i,t}} \pi_{i,t} = p_{i,t}^e q_{i,t} - e - dq_{i,t} - \frac{c}{2} q_{i,t}^2 \tag{1}$$

in which a quadratic cost function describes the available production technology on the market. In other words, $q_{i,t} = \frac{p_{i,t}^e - d}{c}$. Market clearing requires that the price should equalize demand and aggregate supply $D(p_t) = \sum_{i \in M_t} q_{i,t}$ which implies

$$p_t = - \sum_{i \in M_t} \frac{p_{i,t}^e - d}{bc} + \frac{a}{b} \tag{2}$$

Expectations are adaptive, with adjustment speed parameters that differ across different firms:

$$p_{i,t}^e = p_{i,t-1}^e + \alpha_i (p_{t-1} - p_{i,t-1}^e) \quad i \in M_t. \tag{3}$$

The financial wealth of each firm updates at the end of period t recursively:

$$W_{i,t} = \begin{cases} W_{i,t-1} (1 + r_D) + \pi_{i,t} & \text{if } W_{i,t-1} \geq 0 \\ W_{i,t-1} (1 + r_L) + \pi_{i,t} & \text{if } W_{i,t-1} < 0 \end{cases} \tag{4}$$

where we allow for different interest rates for bank deposits and loans, $r_D \leq r_L$. There is an upper bound for borrowing, represented by the parameter $\Psi \leq 0$. If a firms' negative wealth exceeds Ψ then the firm goes bankrupt and exits the market:

$$i \in M_t \wedge W_{i,t} < \Psi \Rightarrow i \notin M_{t+1} \tag{5}$$

So much for market exit. At the same time a pool of potential new firms evaluates mean market profits over the last τ periods and if they are positive on average then they may enter with a positive probability, p :

$$i \notin M_t \Rightarrow \Pr \{i \in M_{t+1}\} = \begin{cases} p & \text{if } \frac{1}{\tau} \sum_{j=1}^{\tau} \bar{\pi}_{t-j} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

We also posit that there is an upper bound for the number of firms that can be in the market, m , which simply means that for given demand schedule it is reasonable to assume a finite number of potential suppliers. Put differently, the number of incumbents plus the number of potential entrants equals a constant m .

Observe that there is no way of defining a steady state for the price independent of the number of firms active on the market. Achieving a steady state for the number of firms would require that there be no potential entrants left and that none of the incumbent be facing bankruptcy. Otherwise a steady number of firms would require that there be some potential entrants who remain outside due to the violation of the

profitability condition in (6). But if that is the case in the longer run some of the market’s suppliers will go bankrupt thus decreasing such number. Notice also that zero is not a steady state for the number of firms, in that it would imply zero profits and therefore a positive probability of entry, again due to (6).

One aspect that needs to be highlighted is the comparison between the model with endogenous participation with its baseline underlying counterpart without entry and exit. In particular the underlying baseline model has stability properties which have been studied by the authors in a previous work (Colucci and Valori 2011). In a nutshell, in cobweb with n firms, what determines whether the model’s steady state, p^* , will be stable or unstable is the product of the value of what can be called the behavioral degree of instability, label it $\bar{\beta}$, and the structural degree of instability, δ , where these are defined precisely as follows:

$$\bar{\beta} = \frac{1}{n} \sum_{i=1}^n \frac{\alpha_i}{2 - \alpha_i} \quad \delta = -\frac{S'(p^*)}{D'(p^*)}.$$

In particular p^* is stable if and only if $\bar{\beta}\delta < 1$.¹ In the present context, once the set of active firms is given, both δ and $\bar{\beta}$ are constant. When participation is endogenous instead, δ and $\bar{\beta}$ depend on the number and the particular assortment of firms active on the market, and they obviously do not convey decisive information for the periods ahead. Nonetheless it is interesting to monitor the evolution of $\bar{\beta}$ and δ because such parameters give an indication of where the underlying model without endogenous participation would go at each point in time. In the present context the actual values of δ and $\bar{\beta}$ at each time t will be calculated as follows:

$$\bar{\beta}_t = \frac{1}{\#M_t} \sum_{i \in M_t} \frac{\alpha_i}{2 - \alpha_i} \quad \delta_t = \frac{\#M_t}{bc} \tag{7}$$

where $\#M_t$ is the number of active firms at time t .

Summarizing, we simulated the model for a range of parameters values keeping track of the evolution of prices, number of active firms, profits, and the values of $\bar{\beta}$ and δ . Parametric choices and results are summarized in the next section.

3 Simulations Parameters and Results

3.1 Parameters

For each combination of parameters described below, we performed Monte Carlo simulations with 10,000 repetitions. The random components of the experiments, beside the intrinsic probabilistic entry rule described in (6), are embedded in the following randomly chosen initial values:

¹ Such parameter appears e.g. in Huang (1995), under the name of multiplier of the equilibrium.

- the set of adaptive adjustment speeds, $\alpha_i \quad i = 1, \dots, m$ (generated as an i.i.d. sample of size m from the uniform distribution on the unit interval);
- the initial number of firms (generated as a random integer between 1 and $\frac{m}{4}$);
- initial values for predictions (generated as an i.i.d. sample from the uniform distribution on the interval of meaningful prices, $[0, \frac{a}{b}]$).

The simulation algorithm was written in Matlab.²

3.1.1 Actual Parameters Used

Demand				
T	a	b		
1000	50	0.5		
Time horizon		Demand parameters		

Firms' parameters					
m	c			d	e
100	Range: $\frac{600}{27}, \frac{1000}{27}, \frac{1400}{27}$			0	10
No. of firms		Costs parameters			

Financial context		
r_D	r_L	Ψ
0	0.05	Range: $-20, -10, -5, 0$
Interest rates		Borrowing constraints

Entry parameters	
p	τ
0.05	12
Entry probability	Profitability condition

3.1.2 Comments on the Parameters

The values for the cost parameter c were chosen so as to ensure a certain comparability of our results with previous works where simulations were presented that had $\delta = 2.7$ (e.g. Brock and Hommes 1997). In our context, as the discussion above explains, the value of δ in general varies with t (see (7)), as it reflects the changing composition of the set of active firms. The values chosen for c reflect average values for δ which are respectively higher (when $c = \frac{600}{27}$), approximately equal (when $c = \frac{1000}{27}$) and lower (when $c = \frac{1400}{27}$) than 2.7. The value of p was chosen with a view to average industry entry rates, as documented e.g. in Geroski (1995).³

² The code used is available from the authors upon request.

³ The parameter $\tau = 12$ can be interpreted as a 3-years period as the horizon decisive to make entry decisions (if quarters are used as time units). However, perturbing this parameter had no significant impact on the outcomes.

3.1.3 Details of the Simulations

The pool of firms is given for the whole time horizon, in the sense that a specific set of values for α_i will be used. Firms that go bankrupt disappear but in the future a firm with the same α_i may re-enter the market. Initial forecasts for entrants in periods $t > 1$ are set to equal the last observed price. If total supply exceeds the largest amount that can be demanded (embedded in the parameter a) each firm is supposed to be able to sell only a fraction of what it produced so that the aggregate amount sold equals a . Of course firms have to bear the costs for the entire amount produced, whether or not it is sold.

3.2 Results and Significant Examples

Our main results are summarized in the following pictures.

3.2.1 Effect of the Borrowing Constraint

One remarkable feature is the effect of the borrowing constraint, embedded in the Ψ parameter. A negative, large value for Ψ means that firms can access a relatively large amount of credit from the banks, whereas $\Psi = 0$ means that firms cannot borrow at all. Such parameter had a strong impact on firms and a less strong but equally interesting effect on prices and therefore on consumers. Indeed, Fig. 1 (right panels) shows that a less stringent borrowing constraint has a positive impact on the average number of active firms and a negative impact on profits and mean prices (with a stronger effect on profits). Also, Fig. 2 (left panel) shows that a larger negative value for Ψ decreases the impact of the heterogeneity in the values of the adaptive speeds, α_i .

3.2.2 Effect of Production Cost

By maneuvering the c parameter which determines the quadratic component of production costs, we can influence the structural degree of instability as shown in (7). In fact a higher c has both a direct negative impact on δ and an indirect impact by way of its effect on $\#M$, which in turn affects $\bar{\beta}$ as well. As a result it turned out there are basically three scenarios, with high, intermediate and low value of the underlying degree of structural instability, corresponding to increasing values of c ($c = \frac{600}{27}, \frac{1000}{27}, \frac{1400}{27}$). Its effect can be seen by inspecting the left panels in Fig. 1. Not surprisingly, higher costs translate into higher mean realized prices. Less obvious is the fact that a higher c also increases the average number of firms that are active over the simulation's time span. An intuition we can provide to explain this is as follows. A higher quadratic cost component will reduce quantities produced by each firm; this in turn, given the linearly decreasing demand, ensures enough profits to the producers so that a certain number of new firms find it profitable to enter. It is also quite interesting to note that the way this effect influences mean profits of producers, depends on the saliency of the borrowing constraint. When no or little borrowing is possible more costs imply less profits, but the implication reverses when firms can access credit more easily.

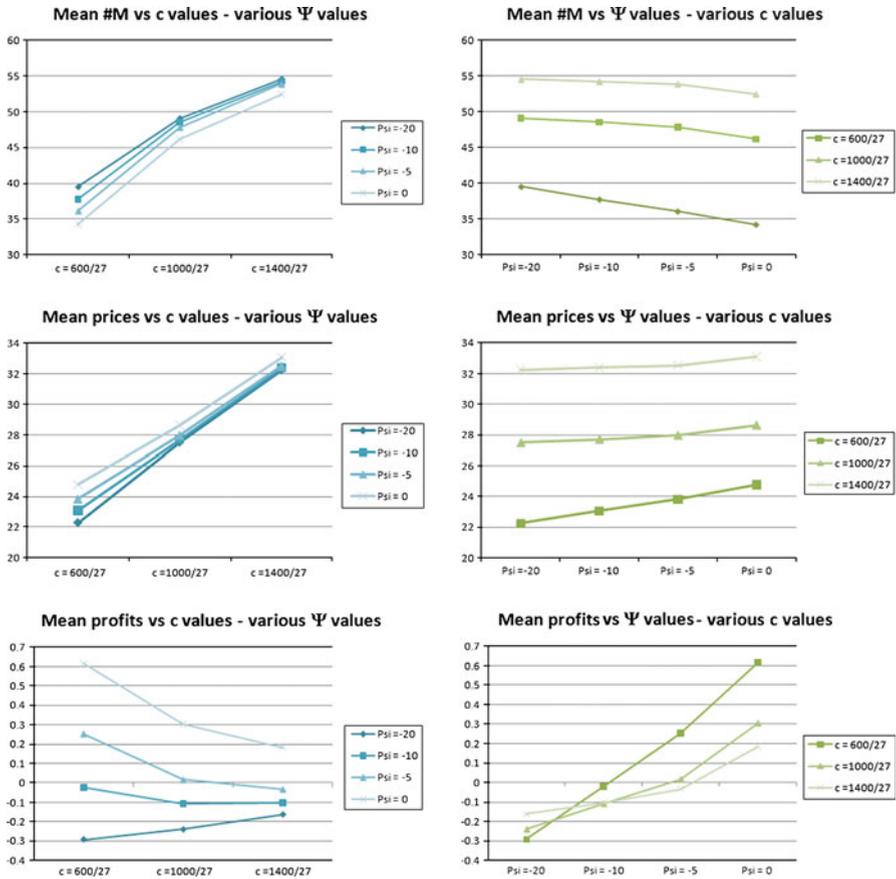


Fig. 1 Number of active firms, prices and profits

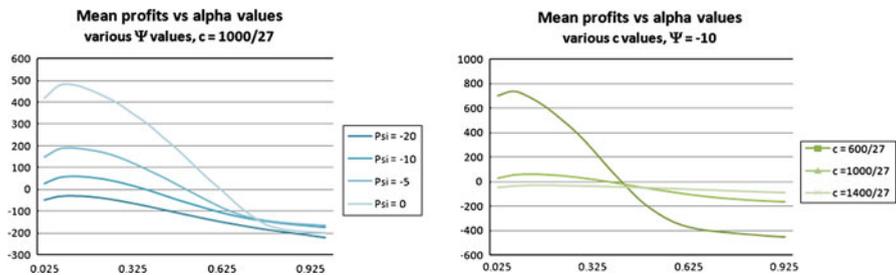


Fig. 2 Impact of expectations parameters on profits

This is a consequence of the reduced scale of the firms under higher cost structure, coupled with the fact that, with larger borrowing possibilities profits become, on average, negative. A discussion of this type appears in Huang (1995), where a higher δ (which governs local stability) associates with higher mean profits. In the present

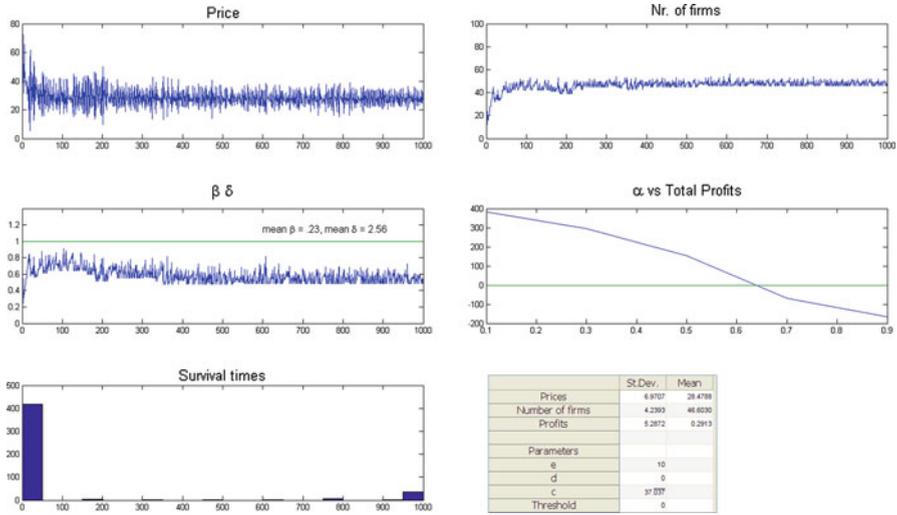


Fig. 3 Example with no borrowing

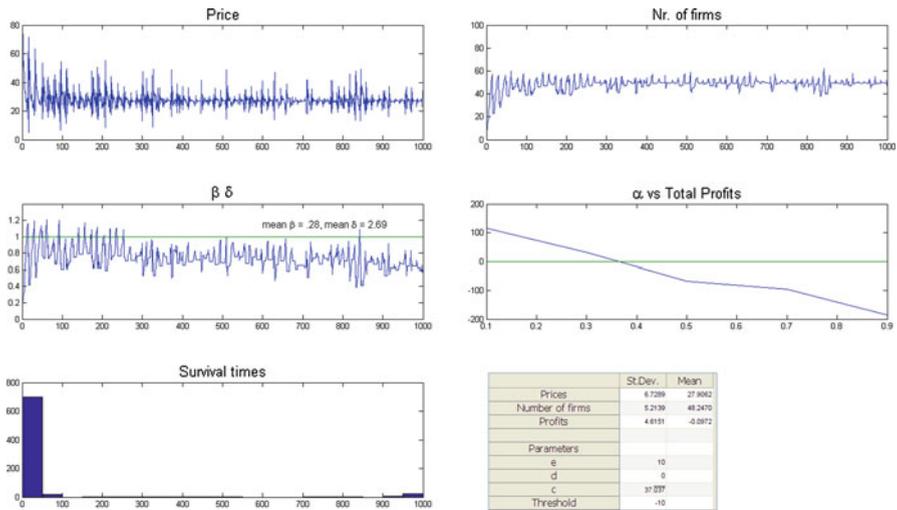


Fig. 4 Example with moderate borrowing constraint

context there is no way to do such comparative statics analysis, given that no steady state exists any more and that both number and types of active firms affect price variability. Notice however that Figs. 3, 4 and 5 (bottom-right table) suggest a positive correlation between price variability (as measured by the standard deviation) and mean profits. A further element which emerges here, where expectations are heterogeneous, is that gains are not evenly distributed among all producers, as explained in the following.

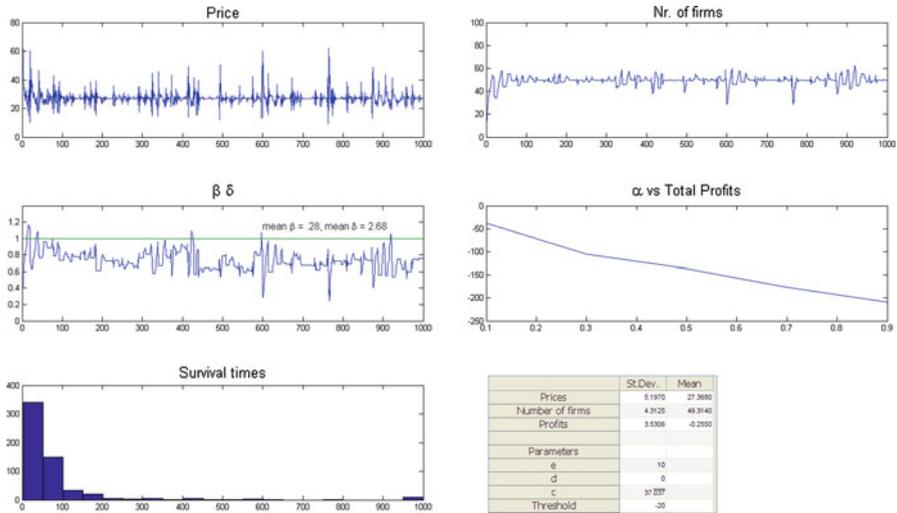


Fig. 5 Example with weak borrowing constraint

3.2.3 When Does Heterogeneity Matter?

Observe (see Fig. 2, right panel) that the effect of heterogeneity in predictors used by the firms is most apparent for relatively higher values of the structural degree of instability (i.e. for lower values of c). So when the environment is intrinsically unstable then the behavioral differences among firms are most relevant. Indeed, such differences are the key to explain the fact that no explosive paths arise, in the sense that the evolutionary mechanism implies that firms with higher α_i (which trigger behavioral instability) are more likely to leave the market due to their lower profitability. Also, as noted above, the amount of credit accessible is also relevant, in the sense that heterogeneity weighs more when constraints are tighter. The fact that on average firms with a smaller value for the adaptive adjustment speed are those that make more profits is an interesting property of the model and is indeed related to an analogous feature we detected in the data generated by the experimental Cobweb economy described in Hommes et al. (2000) as documented in our previous paper Colucci and Valori (2006).

A closer look at the model outcomes can be given by inspecting some examples, depicted in the following figures.

3.2.4 Price Dynamics

The examples show that the evolution in the number of active firms, through market entry and exit, drives an interesting price dynamics, which in some cases shows volatility clusters with periods of booms and busts. This is a promising output of the model, given that prices in certain markets (e.g. food prices) typically show qualitative features of this sort (see the initial discussion in Gouel (2010)). At the same time the

observed dynamics of the price remains bounded, even when the underlying model with exogenous participation would imply explosive paths. This can be seen clearly in the mid-left panel of Fig. 4, for example, which shows that in the first 250 periods the product $\bar{\beta}_t \delta_t$ is often larger than one. Notice moreover the fact that in those initial periods there is also more volatility in the number of active firms as well as in the price.

3.2.5 Survival Times

Another aspect worth remarking is the pattern of average firms' survival times: in general many firms will fail very early in their lives, with a limited number of firms that survive up to T .⁴ Such feature echoes quite closely some of the stylized facts about entry and survival rates put forward by the empirical literature (see Geroski (1995)).⁵ In particular the fact that the capacity of entrants to stay in the market is modest compared to the gross entry rates. Finally observe that, when firms can access credit in excess of their capacity to immediately repay (see Fig. 5, bottom-left panel, where a significant number of cases moves right of the leftmost bar) a relatively higher fraction of firms will live past $t = 50$. Therefore a less tight borrowing constraint will have a significant positive impact on survival times.

4 Conclusions

This paper introduces a mechanism of endogenous participation within the context of a linear cobweb model with adaptive, heterogeneous producers. One of the purposes of the model is that of taking seriously some of the critical arguments that have been raised towards the cobweb literature, in particular those aimed at the usually poor modelling of the financial viability of the firms. So firms face a borrowing constraint and can go bankrupt. At the same time when average profits are positive there is an inflow of new entrants in the market.

The paper's main result is that when the pool of producers is endogenous, the dynamics of prices becomes interesting without the need to resort to nonlinearities. In fact, such dynamics is driven by the mechanism of entry and exit of firms. The presence of average positive profits is an incentive for new firms to enter the market; a larger pool of producers however will typically curb profits, eventually forcing some players to leave. Without endogenous participation the model would be entirely standard and its linearity properties would leave no room for behavior other than convergence to the steady state, explosive paths or 2-period cycles. A number of additional

⁴ This observation proceeds from the bottom-left panels of Figs. 3, 4 and 5, where the histogram of the survival times of all the firms that take part in the simulation (i.e. that are active for at least one time period) is depicted.

⁵ Geroski (1995, pp. 423–424) has: "Stylized fact 3. Entry and exit rates are highly positively correlated, and net entry rates and penetration are modest fractions of gross entry rates and penetration.

....

Stylized fact 4. The survival rate of most entrants is low, and even successful entrants may take more than a decade to achieve a size comparable to the average incumbent."

insights can be gained by inspecting the numerical simulations of the model. A first outcome is that the saliency of the borrowing constraint on firms has a large impact on profits and ultimately on firms' survival chances. While this is in line with findings of Commendatore and Currie (2008), in our scenario when no or little borrowing is possible firms find it harder to survive periods of losses. Besides, tighter constraints have an effect on average prices and therefore on consumer surplus. Second, cost parameters also affect both prices and profits, in a way that depends nontrivially on the prevailing borrowing constraint. Similarly to Huang (1995) we observe a positive correlation between price variability and firms' mean profits. The profits however are not evenly distributed among the producers: on average, firms with a smaller value for the adaptive adjustment speed make more profits. This property is consistent with previous experimental evidence described in Colucci and Valori (2006). Further, the model confirms that behavioral heterogeneity, even in the mild form assumed here, matters, and is in fact crucial to ensure bounded price dynamics. Finally, the model is capable of generating reasonable (given the stylized facts accepted by the empirical literature) patterns of firms survival times.

As an attempt to make an initial step toward incorporating endogenous participation within the cobweb model, this paper naturally leads to a number of additional research questions. Theory-wise on one hand, there is the problem of coming up with tractable versions of the computational model described in the present paper. On the other hand the computational model should be carried further on, e.g. developing a more detailed banking system and exploring different assumptions regarding market entry.

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