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Abstract

We examine within a life-cycle set-up the simultaneous choice of health care and retirement (together with consumption), when health care contributes to both a reduction in mortality and in morbidity. Health tends to impact on retirement via morbidity, determining the disutility of work, and through longevity, determining the need to accumulate retirement wealth. In contrast, the age of retirement drives health through changes in the value of survival and the value of morbidity reductions. We apply our

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model to analyse the effects of moral hazard in the annuity market: While moral hazard always induces excessive health investments and an excessive duration of working life it also triggers an excessive level of consumption if the impact of health on the disutility of work is sufficiently large. We examine a transfer scheme and mandatory retirement as policies to curtail moral hazard. Numerical analysis illustrates the working of our model.

Keywords: annuities, demand for health, moral hazard, life-cycle-model, optimal control, retirement, value of life.
JEL classification: D91, I12, J26

1 Introduction

It is well known that population ageing has significant repercussions for the funding of retirement pensions. Increasing longevity implies that more pension funding needs to be accumulated for a given age of retirement. At the same time and perhaps more importantly, declining fertility rates imply for pay-as-you-go (PAYG) funding a widening gap at aggregate level between contributions and benefits. Reform proposals range from parametric adjustments in the pension scheme to a shift from PAYG to a funded pension system (see e.g. Disney 2000), but they typically include as a corner stone an increase in the retirement age.

This brings into focus the role of health and health care: On the one hand, if individuals are expected to work longer productively, this requires they are still in shape to do so. On the other hand, significant changes in the length of the working life may alter the individual's behaviour towards their health. More generally, the increase in longevity as one of the factors causing the need to readjust the retirement scheme is determined by the individual's health and health-related behaviour. The link between retirement and health at individual level has received considerable attention in the empirical literature. However, this is not reflected in theoretical work. While there are first efforts at gaining theoretical insights into the relationship between health and retirement (Bloom et al. 2007, Galama et al. 2008, Sheshinski 2008), the health-longevity-retirement nexus remains to large extent underexplored.

The relationship between health and retirement arises through (at least) four channels: (i) the positive relationship between health and longevity; (ii) the positive impact of health on productivity; (iii) the impact of work on health; and (iv) the impact of work-related income (i.e. earnings) on health. We briefly consider these effects in turn: (i) By driving increases in longevity, improvements in health trigger a need for accumulating greater retirement wealth and thus a need for an increase in the retirement age. (ii) By raising productivity (and/or lowering the disutility of work) health improvements allow the individual to earn a higher income over a fixed length of working life. While the wealth effect may allow the individual to retire earlier, improvements in productivity imply a substitution effect that calls for a postponement of retirement, leaving the overall effect ambiguous. (iii) The supply of labour is likely to have a direct impact on health, the direction of which depends on the working environment. If work is physically demanding, stressful and/or dangerous, the impact of labour supply on health is negative.

Thus, (an early) retirement would lead to a reduced depreciation of health (possibly but perhaps less likely even to improvements in health). On the other hand, if the supply of labour allows an individual to 'stay in shape', the impact of retirement on health may, in fact, be negative. (iv) Increases in life-time labour income coming with a later retirement allow the individual to consume greater amounts of health care, thus leading to improved health.

A large body of empirical literature has been devoted to disentangling the various strands of relationships between health and retirement. While most of the empirical evidence supports a positive impact of health on the labour force participation of the elderly (see e.g. Bound et al. 1999, Lindeboom and Kerkhofs 2009, Jones et al. 2010),[1] the evidence is somewhat more controversial for the reverse impact of retirement on health. Using panel data, Kerkhofs and Lindeboom (1997) and Lindeboom and Kerkhofs (2009) find that health tends to deteriorate with employment and labour market history.[2] Bound and Waidmann (2007), Neuman (2007) and Coe and Zamarro (2011) find positive health effects of pension eligibility (as a measure for retirement that is exogenous to the individual).[3] In contrast, Dave et al. (2008), using panel data; Kuhn, Wuellrich and Zweimüller (2010), using a natural experiment; and Behncke (2011), using panel data and nonparametric matching, identify significant negative effects of (early) retirement on various measures

[1] There is less agreement on the extent to which retirement decisions are driven by health as opposed to financial incentives, with evidence pointing both at financial incentives (e.g. Bazzoli 1985, French 2005) and at health (Dwyer and Mitchell 1999, McGarry 2004) as the prime driver. The role of financial incentives may well depend on the particular route to retirement as, indeed, on the individual's health state itself. Kerkhofs et al. (1999) find that financial incentives rather than health explain the selection into early retirement schemes but health rather than financial concerns explains the selection into disability insurance or unemployment insurance as a pathway to retirement. Similarly Erdogan-Ciftci et al. (2011) show that financial incentives are effective only conditional on good health.

The literature is also somewhat mixed on the role of a gradual deterioration of health as opposed to health shocks as explanators of retirement: Methodologically, health shocks offer a convenient way of overcoming the problem of endogeneity bias caused by the correlation of measures of health and unobserved heterogeneity (Disney et al. 2006). They are good predictors of the onset of disability and thus of (early) retirement , but as they are rare events, the more gradual deterioration of health must play what is perhaps the more important role in explaining retirement (Lindeboom et al. 2006).

[2] Lindeboom and Kerkhofs (2009) is one of the few papers that estimates jointly both the effect of retirement on health and the effect of health on labour force participation.

[3] Coe and Lindeboom (2008) caution, however, that the anticipation of retirement may bias these estimates to zero.

of health.

A positive link between longevity and the number of life-years spent in good health is empirically well established by now (for a discussion and references see Bloom et al. 2007). According to the 'compression of morbidity' hypothesis the period spent in ill health towards the end of life is compressed (either in the absolute number of life-years or at least in proportion to total life-time). The health improvements over the life course underlying the increase in longevity should also imply that individuals enjoy greater productivity (or lower disability from labour supply). Bloom et al. (2007) employ this relationship within a life-cycle model with endogenous retirement to examine how an exogenous increase in longevity bears on the optimal timing of retirement. They show that the income effect related to higher productivity dominates the substitution effect so that individuals respond to increases in longevity by demanding both more leisure and more consumption. Hence, while individuals tend to increase the length of their working lives (in order to accumulate greater retirement wealth) they do so less than proportionately, implying that a greater share of the lifetime gained is spent in retirement.[4] In a related analysis, d'Albis et al. (2012) examine the impact of greater longevity on retirement behaviour when mortality is age-specific rather than uniform as in Bloom et al. (2007). They show that by raising expected lifetime earnings declines in mortality during the early life-years may trigger a decrease in the retirement age. Neither Bloom et al. (2007) nor d'Albis et al. (2012) consider health a subject of individual choice.

The endogeneity of health is addressed by Galama et al. (2008) who extend the Grossman (1972) model of life-cycle demand for health to allow individual choices of both retirement and health expenditure (besides consumption). Health expenditure contributes towards greater productivity and provides a direct benefit of consumption but does not raise longevity. Indeed, the life span is exogenous and fixed throughout the analysis. Galama et al. (2008) derive a number of different patterns of health, health expenditure, and consumption depending on the (exogenous) onset of retirement. At the point of retirement, health loses its 'productive' value, implying that the desired level of the health stock decreases discontinuously. Hence, according to one scenario, individuals may begin to invest in health at some point prior to retirement in order to maintain their stock of health up to the point of re-

[4]Prettner and Canning (2011) extend Bloom et al.(2007) to a general equilibrium setting.

tiring. Post-retirement they forego health investments up to the point where health has depreciated to the lower optimal level at which (re-)investments become necessary in order to maintain the consumption value of health. In a second stage of analysis the authors solve numerically for the optimal retirement age. These simulations show for instance, that workers who earn a higher base-line wage tend to (re-)invest earlier in their health and tend to retire later. Thus, in their model, and in contrast to Bloom et al. (2007), the substitution effect in the consumption-leisure trade-off is dominant. In contrast, higher levels of initial health induce earlier retirement by a pure income effect, whereas, surprisingly perhaps, a greater deterioration of health induces later retirement (a negative income effect).[5]

While the models by Bloom et al. (2007) and by Galama et al. (2008) complement each other, each of them is omitting an important aspect of the health-retirement-longevity nexus. We therefore propose a unified model allowing for an analysis that embraces the longevity and morbidity effects of health (similar to Bloom et al. 2007) but at the same time endogenises the individual's choice of health care (similar to Galama et al. 2008). In our model, morbidity relates to a higher disutility of labour. Thus, we obtain a richer model that captures some of the feedback of longevity on retirement (as in Bloom et al. 2007) but, at the same time, allows us to analyse how retirement affects the choice of health care. More specifically, we provide a generalisation of the individual willingness to pay for health improvements to account for the individual's retirement and for the value of reductions in the disutility of labour in addition to the well-known value of mortality reductions, i.e. the statistical value of life.[6] Thus, we show under which conditions health investments are complementary to a postponement of retirement. We also examine in detail how changes in health care patterns bear on the individual's retirement incentive.

We apply our model to study the effects of moral hazard within the annuity market, an issue which is addressed neither by Bloom et al. (2007) nor

[5] A fourth paper considering the interrelationship between health and retirement is Hansen (2010). However, in his model entry into retirement, although in principle subject to individual choice, is to large extent enforced through strong incentives within the pension scheme. Moreover, the focus of his work lies more on the impact of missing annuity markets.

[6] Other generalisations of the value of life concept include Birchenall and Soares (2009) and Kuhn, Wrzaczek and Oeppen (2010), accounting for the value of progeny, and Kuhn et al. (2011), accounting for spillovers on mortality in the provision of health care.

by Galama et al. (2008). As is well-known from Davies and Kuhn (1992) and Philipson and Becker (1998) individuals invest excessively into health and longevity when they take annuity returns as given. We follow Sheshinski (2008) in extending this framework to account for endogenous retirement but go beyond his analysis by considering (i) the whole life-cycle pattern of health investments (rather than a singular up-front investment) and (ii) by allowing health care not only to reduce mortality but also to reduce morbidity and, thus, the disutility of labour. We show that while moral hazard always triggers a postponement of retirement, it triggers an excessive level of consumption if the morbidity-effect is strong. This contrasts the 'conventional' finding of too low a level of consumption in the presence of moral hazard when morbidity is not affected (Davies and Kuhn 1992, Philipson and Becker 1998, Sheshinski 2008). In the presence of a morbidity effect, moral hazard not only pulls for a postponement of retirement in order to accommodate the life-cycle costs of excessive health and longevity but also pushes towards a further postponement of retirement as the disutility of labour falls below its first-best level. The additional expansion of the working life may then allow the individual to consume more than in a first-best. While somewhat deceptively moral hazard then induces a longer life, lower disutility of labour and a higher level of consumption, this is nevertheless inefficient as the individual suffers the disutility of an excessive working life. We derive an age-dependent transfer that, in principle, would restore the first-best allocation. However, as such a policy involves the significant taxation of health care in particular for the ages with high spending levels, it stands in stark contrast to real-world health insurance, involving subsidisation rather than taxation of out-of pocket expenses. Thus, we consider the scope for mandatory (early) retirement as a second-best policy. Indeed, on the margin earlier retirement contributes towards an increase in life-cycle utility if by curtailing health expenditure it reduces the cost of moral hazard. This is true even if earlier retirement comes at a (second-order) loss of utility from consumption. We complement our analysis by numerical simulations, which illustrates the life-cycle allocation and the distortions arising from moral hazard.[7]

The remainder of the paper is organised as follows. In section 2 we set out the life-cycle model and study the optimal allocation. Section 2.1 derives

[7]Cremer et al. (2004, 2006) derive early retirement as a second-best policy in an adverse selection context. While Cremer et al. (2006) allow health expenditure to reduce the disutility of labour, as we do, longevity is exogeonus and longevity-related moral hazard does not matter. Thus, the problem they study is of a rather different nature.

the first-order conditions; section 2.2 derives the value of health; section 2.3 studies the structure and dynamics of the optimal allocation; while section 2.4 examines the complementarity between health, retirement and consumption. In section 3 we analyse the distortions arising from moral hazard in the annuity market, while in section 4 we study mitigating policies. Section 5 illustrates the workings of our model by way of a numerical analysis and section 6 concludes.

2 The model

Consider an individual who chooses age-specific consumption, $c(t)$, health care, $h(t)$, and the age of retirement, τ, to maximise lifetime utility. The life-course falls into two distinct phases, separated by τ: working life and time in retirement. Similar to Bloom et al. (2007) we assume that in each period the individual enjoys utility from consumption $u(c(t))$. The period utility from consumption is increasing and concave: $u_c > 0$, $u_{cc} \leq 0$. In addition, we assume $u(0) \geq 0$ and $u_c(0) = +\infty$. During her working life the individual suffers a disutility $\nu(\cdot)$ from providing a fixed quantity of labour.[8]

Before providing further detail on the disutility of labour we turn to a description of health and survival. In each period the individual faces a mortality risk, where the survival probability evolves according to the age specific mortality rate, $\mu(t, h(t))$, which depends on the current age and the health investment. The corresponding state equation is:

$$\dot{S}(t) = -\mu(t, h(t))S(t), \quad S(t_0) = 1, \tag{1}$$

where t_0 indicates the birth date. We assume that the mortality rate $\mu(t, h(t))$ satisfies

$$\begin{aligned} \mu(t, h(t)) &\in (0, \tilde{\mu}(t)] \ \ (\forall t); \quad \mu(t, 0) = \tilde{\mu}(t), \ \mu(t, \infty) \geq 0 \ (\forall t) \\ \mu_h(\cdot) &< 0, \ \mu_{hh}(\cdot) > 0; \quad \mu_h(t, 0) = -\infty, \ \mu_h(t, \infty) = 0 \ (\forall t), \end{aligned}$$

where $\tilde{\mu}(t)$ is the "natural" mortality rate resulting without any health care. Hence, by purchasing health care and lowering the instantaneous mortality

[8]This assumption is standard in many life-cycle models with retirement (see e.g. Kingston, 2000; Bloom et al, 2007; Sheshinski, 2008; Heijdra and Romp, 2009; d'Albis et al, 2012; Kalemli-Ozcan and Weil, 2010).

rate an individual can improve its survival prospects but only so at diminishing returns. At this point we note that we can (re-)interpret $S(t)$ as the individual's age-specific stock of health, where (1) then describes the development of the stock of health over the life-course.[9] Similar to the models by e.g. Grossman (1972), Ehrlich and Chuma (1990), Ehrlich (2000) or Galama et al. (2008) the health stock is subject to depreciation. Health care allows to reduce the depreciation but (in contrast to the models a la Grossman 1972) not to reverse it. In our model the demand for health depends on survival prospects and, thus, on the stock of health, but it is importantly shaped by the impact of health on mortality (=depreciation), $\mu_h(\cdot)$. Indeed, as we will show below (see sections 2.3 and section 5) our model is compatible with an inversely U-shaped age-profile of health expenditure despite an ongoing reduction of the health stock.[10] Regarding the irreversibility of health decline, we should stress that our model is written from an ex-ante rather than an ex-post perspective. We do not have in mind the treatment of certain specific illnesses, leading to a recovery and, thus, to an increase in the health stock, but rather more broadly the incentives to shape the long-run development of health and mortality. Here, the long-run depreciation of health is documented not only by declining survivorship (in fact, typically at increasing rates) but also by the gradual accumulation of health deficits (for an overview see Strulik 2010: section 6).

We assume a health-dependent disutility of work $\nu\left(t, S(t)\right)$. More specifically, we assume that at any age t the disutility $\nu\left(t, S(t)\right)$ declines in the stock of health $S(t)$ but at a decreasing rate, i.e. $\nu_S \leq 0$, $\nu_{SS} \geq 0$.[11] The degree to which the disutility of work is health dependent will play a prominent role for our subsequent analysis. Intuitively, we would expect the dependency

[9]This is representative of a more general positive relationship between the stock of health and survival. Note that such a relationship is broadly in line with the compression of morbidity hypothesis.

[10]Our model therefore does not give rise to a positive relationship between the stock of health and expenditure as in Grossman (1972), which is at odds with empirical evidence. Galama et al. (2008) provide a different rationale as to why health expenditure falls with the stock of health. In their model, following Wolfe (1985), an individual does not demand any health care up to the point that natural depreciation has eroded a stock of health, which initially is large, to the level at which positive health investments become profitable.

[11]Bloom et al. (2007) employ this argument when assuming that disutility of work increases with age and falls with the life expectancy at birth. Our model captures both aspect - poorer health with the progression of age and the compression of morbidity coming along with an increase in life expectancy - with one and the same variable.

to vary strongly with the individual's occupation. For blue collar jobs, involving heavy labour, the disutility of labour is prone to increase strongly when health is deteriorating, but this relationship may be much weaker for many clerical occupations. Thus, we will allow ν_S to vary in absolute value, including the special case where $\nu_S = 0$. In order to ensure even for this particular case a unique retirement age, we assume $\nu_t \geq 0$, $\nu_{tt} \geq 0$.[12]

Consumption, health and savings are financed out of earnings $w(t)$ during the first (working) phase of life. We assume that earnings are bounded and exogenously given. In order to sustain consumption and health during the retirement phase the individual invests in annuities (Yaari 1965). We assume that in contrast to e.g. Hurd (1989) and Leung (1994, 2007) full availability of annuities to the individual; and in contrast to e.g. Brunner and Pech (2008) actuarially fair pricing of annuities. This notwithstanding, annuity prices are typically based on expected or average mortality $\bar{\mu}(t)$, as observed in life-tables, and not on the individual's current mortality $\mu(t, h(t))$. The individual will then take the return on annuities $r + \bar{\mu}(t)$ as given when choosing the level of health care. As Davies and Kuhn (1992), Philipson and Becker (1998) and Sheshinski (2008) show, this implies health-related moral hazard, where individuals tend to invest too much into longevity. In order to study the implications of such an imperfection in the annuity market, we consider a hypothetical annuity market, paying a return of $r + \theta\bar{\mu}(t) + (1-\theta)\mu(t, h(t))$ with $\theta \in [0, 1]$. The polar case $\theta = 0$ then corresponds to a (hypothetical) market in which the return to 'individualised' annuities responds instantaneously to changes in individual health care, whereas the case $\theta = 1$ describes the (realistic) market with moral hazard in which the individual takes annuity prices as given. This not withstanding, the annuity market is actuarially fair, implying that the market equilibrium obeys $\bar{\mu}(t) = \mu(t, h(t))$.

We therefore obtain the following two stage budget equation where we assume that the individual starts and ends with zero assets.

$$\begin{aligned}
\dot{A}(t) &= w(t) - c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(t_0) = 0 \quad \text{for } t \leq \tau \\
\dot{A}(t) &= -c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(T) = 0 \quad \text{for } t \geq \tau
\end{aligned}$$

Here, T denotes a point in time at which the individual has perished

[12] For the general case with $\nu_S < 0$, the disutility of labour is increasing with age due to the decline in health (1). In this case, we could set $\nu_t = \nu_{tt} = 0$.

with certainty.[13] Applying a discount rate ρ, we can formulate the following dynamic optimisation problem with state variables $A(t), S(t)$ and control variables $c(t), h(t), \tau$:

$$\max_{c(t),h(t),\tau} \int_{t_0}^{\tau} e^{-\rho t} S(t)(u(c(t)) - \nu(t, S(t)))dt + \int_{\tau}^{T} e^{-\rho t} S(t) u(c(t))dt \qquad (2)$$

subject to

$$\begin{aligned}
\dot{A}(t) &= w(t) - c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(t_0) = 0 \quad \text{for } t \leq \tau \\
\dot{A}(t) &= -c(t) - h(t) + (r + \theta\bar{\mu} + (1-\theta)\mu)A(t), \quad A(T) = 0 \quad \text{for } t \geq \tau \\
\dot{S}(t) &= -\mu(t, h(t))S(t), \quad S(t_0) = 1
\end{aligned}$$

Here, the first-integral of the objective function denotes the expected present value of the utility stream over the working life, while the second integral denotes the expected present value of the utility stream during retirement. The constraints are given by the (two part) budget equations, the movement of survivorship/health capital as well as the initial and end point conditions. Note that the model with $\theta = 0$ generates a first-best solution without moral hazard, whereas the model with $\theta = 1$ generates the second-best with moral hazard.

2.1 The optimal solution

The following section presents the derivation of the optimal solution while the subsequent sections offer an intuitive explanation of the optimal health and consumption paths.

To solve the problem in (2) we apply the optimality conditions for two-stage optimal control problems as outlined in Grass et al. (2008, pp. 386). Using the current value Hamiltonians for the first and second periods

$$\begin{aligned}
\mathcal{H}^1 &= Su(c) - S\nu(S) - \lambda_S^1 \mu(h)S + \lambda_A^1(w - c - h + (r + \theta\bar{\mu} + (1-\theta)\mu)A) \\
\mathcal{H}^2 &= Su(c) - \lambda_S^2 \mu(h)S + \lambda_A^2(-c - h + (r + \theta\bar{\mu} + (1-\theta)\mu)A)
\end{aligned}$$

[13] Although we have assumed a finite time horizon, the model structure also allows to choose $T = \infty$. In that case the end condition $A(T) = 0$ would have to be replaced by $\lim_{t\to\infty} A(t) = 0$ and the transverslity conditions of the adjoint variables would have to be replaced by appropriate ones. The optimal allocation as described in the next section would not change qualitatively.

where λ_S^i and λ_A^i, $i = 1, 2$, denote the adjoint variables relating to S and A, respectively, we obtain the following set of first order conditions for period $i = 1, 2$ (at an inner optimum)

$$\mathcal{H}_c^i = Su_c(c) - \lambda_A^i \stackrel{!}{=} 0, \tag{3}$$

$$\mathcal{H}_h^i = -\mu_h\big(S\lambda_S^i - A\lambda_A^i(1-\theta)\big) - \lambda_A^i \stackrel{!}{=} 0. \tag{4}$$

The two conditions determine optimal consumption and health investments, respectively. For the adjoint equations $i = 1, 2$ we obtain

$$\begin{aligned}\dot{\lambda}_S^1 &= (\rho+\mu)\lambda_S^1 - (u(c) - \nu(S) - S\nu_S(S)),\\ \dot{\lambda}_S^2 &= (\rho+\mu)\lambda_S^2 - u(c),\\ \dot{\lambda}_A^i &= (\rho - r - \theta\bar{\mu} - (1-\theta)\mu)\lambda_A^i.\end{aligned}$$

To account for the switching instant at the age of retirement, the following matching conditions for the adjoint variables

$$\begin{aligned}\lambda_A^1(\tau) &= \lambda_A^2(\tau) =: \lambda_A^\tau\\ \lambda_S^1(\tau) &= \lambda_S^2(\tau) =: \lambda_S^\tau\end{aligned}$$

and the Hamiltonian $(\mathcal{H}^1(\tau) = \mathcal{H}^2(\tau))$[14]

$$\frac{\nu(\tau, S(\tau))}{u_c(c(\tau))} = w(\tau) \tag{5}$$

must hold. The latter condition determines the optimal age of retirement. The following Lemma establishes a set of sufficient conditions for the existence of a unique (and interior) age of retirement (see Appendix A for a proof).

Lemma 1 *An interior solution to (5) exists if*

E1) $\frac{\nu(t_0,1)}{u_c(c(t_0))} < w(t_0)$

E2) $\lim_{t\to T}\nu(t, S(t)) = +\infty$ *or* $\frac{\nu(T,S(T))}{u_c(c(T))} > w(T)$

[14] The general matching conditions are more complicated as they allow for switching costs and also the fact that the first and/or second period objective might depend on the switching time (cf. Grass et al. 2008, p. 387).

are satisfied. The resulting solution $\tau \in (t_0, T)$ is unique if

U1) $\rho \geq r$

U2) $w'(t) \leq 0$ *or* $w'(t) > 0$, $w''(t) \leq 0$

Before turning to an interpretation of the optimality conditions, we introduce the value of health as a convenient measure for our further analysis.

2.2 Value of Health

We can calculate the value of health as the willingness to pay for a small reduction of the mortality rate - or equivalently the depreciation rate on the health stock - at age t. Conceptually, the value of health is identical to the value of life, as was first developed in a formal manner by Shepard and Zeckhauser (1984) (see also Rosen 1988, Ehrlich and Chuma 1990, Ehrlich 2000, Johansson 2002, Murphy and Topel 2006). Denoting by V the value function corresponding to the optimisation problem in (2), we define the value of health (VOH) as

$$\psi^i(t) := -\frac{\partial V/\partial \mu}{\partial V/\partial A} = \frac{\lambda^i_S S - (1-\theta)\lambda^i_A A}{\lambda^i_A} = \frac{\lambda^i_S}{u_c} - (1-\theta)A \tag{6}$$

where the third equality follows from the first-order condition (3). Integrating the adjoint equation $\dot{\lambda}^i_S$ and the budget equation, and substituting λ^i_S and A, we obtain the following expression for the VOH[15] where $\Phi(s,t) := e^{-r(s-t)}\frac{S(s)}{S(t)}$ denotes the discounted survival probability to age s conditional on surviving to age t.

$$\begin{aligned}
\psi^1(t) \quad : \quad &= \psi(t \leq \tau) = \int_t^\tau \Phi(s,t)\frac{u\left(c\left(s\right)\right) - \nu\left(S\left(s\right)\right)}{u_c\left(c\left(s\right)\right)}ds \\
&+ \int_\tau^T \Phi(s,t)\frac{u\left(c\left(s\right)\right)}{u_c\left(c\left(s\right)\right)}ds + (1-\theta)\left[H\left(t\right) - E\left(t\right)\right] \\
&- \int_t^\tau \Phi(s,t)\frac{S(s)\nu_S}{u_c\left(c\left(s\right)\right)}ds \qquad (7) \\
\psi^2(t) \quad : \quad &= \psi(t \geq \tau) = \int_t^T \Phi(s,t)\frac{u\left(c\left(s\right)\right)}{u_c\left(c\left(s\right)\right)}ds - (1-\theta)E\left(t\right), \qquad (8)
\end{aligned}$$

[15] It is easy to check that $\psi^1(\tau) = \psi^2(\tau)$. Thus, the VOH also satisfies a matching condition at the switching point.

where

$$H(t) \;:=\; \int_t^{\tau} \Phi(s,t) w(s)\, ds \tag{9}$$

$$E(t) \;:=\; \int_t^{T} \Phi(s,t) \left(c(s) + h(s)\right) ds \tag{10}$$

denote, respectively, human wealth, i.e. income over the remaining life course, and expenditure over the remaining life course. The VOH during the working phase can then be decomposed into five components corresponding to the terms on the RHS of (7): (i) The gross value attached to the remaining working life, consisting of the (discounted) gross value of each year of working life $\frac{u-\nu}{u_c}$ summed over the individual's remaining working life. (ii) The gross value attached to the retirement phase, amounting to the (discounted) gross consumer surplus $\frac{u}{u_c}$ aggregated over the remaining life-course. (iii) Human wealth $H(t)$ and (iv) expenditure over the remaining life-course, $E(t)$, which are weighted by $1-\theta$ and, therefore, count towards the VOH only within a first-best market with individualised annuities ($\theta = 0$) but not in a second-best market with longevity-related moral hazard ($\theta = 1$). (v) the aggregate value of reductions in the disutility of work, $-\frac{S\nu_S}{u_c} > 0$, resulting from health improvements over the remaining working life.

Note that (i)-(iv) correspond to the value of survival, as is typically embraced by the value of a statistical life as in the literature referenced earlier.[16] Obviously, the value of survival refers to reductions in mortality. In contrast, (v) can be understood as the value assigned to reductions in morbidity. Equation (8) denotes the VOH after entry into retirement, where the disutility of work, the value of work-related improvements in health and human wealth are no longer relevant.

[16]Typically this literature does not account for retirement.

2.3 Structure and dynamics of the optimal allocation

Combining (3) and (4), observing that $\lambda_A^i(t) = \lambda_A^i(s)\, e^{-\rho(s-t)}\Phi(t,s)$, and using (6) we can express the optimality conditions for $\{c^*(t), h^*(t), \tau^*\}$ as

$$\frac{u_c(c^*(t))}{u_c(c^*(s))e^{\rho(s-t)}} = e^{r(s-t)}, \tag{11}$$

$$-\frac{1}{\mu_h(h^*(t))} = \psi^i(t), \quad i=1,2, \tag{12}$$

$$\frac{\nu(\tau^*, S(\tau^*))}{u_c(c^*(\tau^*))} = w(\tau^*). \tag{13}$$

The distribution of consumption over the life-cycle is determined by the familiar Euler equation (11), equating the marginal rate of intertemporal substitution with the compounded interest. Note that we can then write consumption

$$c^*(t) = c_0 e^{(r-\rho)(t-t_0)}$$

as a function of the baseline-consumption level, c_0, time and the interest vs. discount rate. Health investments are chosen such that the effective cost of saving a life-year equals the VOH. While the optimal distribution of consumption is independent of whether or not the individual is retired, this is not the case for health care. Finally, given that the conditions in Lemma 1 are satisfied, retirement occurs at the point where the monetary disutility of working just equals the earnings. Indeed, as Lemma 1 shows, if the wage rate exceeds (falls short of) the monetary disutility it is optimal to remain in employment (to retire), a familiar optimality condition for retirement (see e.g. Bloom et al. 2007).

From the first order condition (11) we can derive the time path of consumption (omitting time indices)

$$\dot{c}^* = \frac{u_c(c^*)}{u_{cc}(c^*)}(\rho - r - \theta(\bar{\mu} - \mu)) = \frac{u_c(c^*)}{u_{cc}(c^*)}(\rho - r),$$

which is the dynamic representation of the Euler equation. Note that the second equality follows from the fact that in equilibrium $\mu = \bar{\mu}$ holds. Thus, the consumption path, but not the level of consumption, remains unaffected by moral hazard in the annuity market. As is common, consumption increases (decreases) over the life-course if and only if the discount rate falls short of (exceeds) the interest rate. From (12) we obtain the time path of health investments

$$\dot{h}^* = -\frac{\mu_{ht}(h^*)}{\mu_{hh}(h^*)} - \frac{\mu_h(h^*)}{\mu_{hh}(h^*)} \left[\begin{array}{c} r + \mu(h^*) \\ -\frac{1}{\psi^1(t)} \left(\frac{u-\nu}{u_c} - \frac{\nu_S S}{u_c} + (1-\theta)(w - c^* - h^*) \right) \end{array} \right],$$

for $t \leq \tau^*$, and

$$\dot{h}^* = -\frac{\mu_{ht}(h^*)}{\mu_{hh}(h^*)} - \frac{\mu_h(h^*)}{\mu_{hh}(h^*)} \left[r + \mu(h^*) - \frac{1}{\psi^2(t)} \left(\frac{u}{u_c} + (1-\theta)(-c^* - h^*) \right) \right].$$

for $t > \tau^*$. The evolution of health investments is determined by two forces: (i) The change in the marginal effectiveness of health investments with the progress of age, corresponding to the first term on the RHS of both branches of the path. Recalling that $\mu_{hh} > 0$, i.e. decreasing returns to health care at any given age, it follows that health investments tend to increase directly with age as long as the marginal effectiveness of health care increases with age, $\mu_{ht} < 0$. Typically, this tends to be true for ages up to the 60s and 70s with health expenditure having little impact at young ages before the onset of life-threatening conditions. For the highest ages health expenditure is likely to become less effective in combatting mortality so that $\mu_{ht} > 0$. (ii) The change over the life-course in the VOH. Indeed, it can be verified that the second term on the RHS equals the rate of change in the respective VOH $\frac{\dot{\psi}^i(t)}{\psi^i(t)}$. On the one hand, the VOH increases with the effective interest rate $r + \mu$; on the other hand, it falls as with each passing life year the value of this year is written off.[17] During the working life ($t \leq \tau^*$), this depreciation includes the gross value of a life year, $\frac{u-\nu}{u_c}$, the value of morbidity reduction, $-\frac{\nu_S S}{u_c}$, while the net savings $w - c^* - h^*$ are (fully) included if and only if annuities are individualised ($\theta = 0$). For the retirement phase ($t > \tau^*$) the value of a life years passed excludes the disutility of working, earnings and the value of morbidity declines, $-\frac{\nu}{u_c} + w - \frac{\nu_S S}{u_c}$. Recalling that at the point of optimal retirement, τ^*, we have $-\frac{\nu}{u_c} + w = 0$, it follows that at this point the depreciation of the VOH falls discontinuously by an amount $-\frac{\nu_S S}{u_c} > 0$, corresponding to the reduction in morbidity, which is no longer valued during retirement. Thus, with the onset of retirement health investments may suddenly increase at a larger pace; decline at a smaller pace;

[17]Recall here that $-\frac{\mu_h(h)}{\mu_{hh}(h)} > 0$.

or indeed relapse from a decline back into an increase (see Figure 4 in section 5).

2.4 Life-cycle complementarity

This sub-section explores the complementarity between health and consumption, health and retirement; and retirement and consumption. While of some interest in their own right, these relationships will subsequently help us in establishing our main result. Starting with health we note from (12) that the partial effect of a variable $x \in \left\{c_0, \tau, \theta, h(\widehat{t}) \left|_{\widehat{t}\neq t}\right.\right\}$ on health care $h^*(t)$ is given by

$$\frac{\partial h^*(t)}{\partial x} = \frac{\mu_{hh}}{\mu_h^2}\frac{\partial \psi^i(t)}{\partial x},$$

implying that $sgn\frac{\partial h^*(t)}{\partial x} = sgn\frac{\partial \psi^i(t)}{\partial x}$. Hence, the dependency of $h^*(t)$ on a variables x is conveniently described by the effect of x on VOH. We can summarise these effects as follows (see Appendix B for a proof).

Lemma 2 *(i) Forward complementarity to (future) health:*

$$\frac{\partial \psi^i(t)}{\partial h\left(\widehat{t}\right)}\left|_{\widehat{t}\in(t,T)}\right. = -\Phi(\widehat{t},t)\left[\mu_h\left(\widehat{t}\right)\psi^i(\widehat{t}) + 1 - \theta\right] = \theta\Phi(\widehat{t},t) \geq 0 \text{ for } i = 1, 2.$$

(ii) Backward complementarity to (past) health: $\frac{\partial \psi^2(t)}{\partial h\left(\widehat{t}\right)}\left|_{\widehat{t}\in(t_0,t)}\right. = 0$ *and*

$$\frac{\partial \psi^1(t)}{\partial h\left(\widehat{t}\right)}\left|_{\widehat{t}\in(t_0,t)}\right. = \mu_h\left(\widehat{t}\right)\int_t^\tau \Phi(s,t)\frac{\left[2\nu_S + \nu_{SS}S\right]S}{u_c}ds \geq 0$$

if $2\nu_S\left(s, S\left(s\right)\right) + \nu_{SS}S\left(s\right) \leq 0$ for all $s \in [t, \tau]$.

(iii) Complementarity to consumption:

$$\begin{aligned}\frac{\partial \psi^2(t)}{\partial c_0} &= \int_t^T \Phi(s,t)\left(\frac{-uu_{cc}}{u_c^2} + \theta\right)e^{(r-\rho)(s-t_0)}ds > 0\\ \frac{\partial \psi^1(t)}{\partial c_0} &= \int_t^\tau \Phi(s,t)\left[\frac{-\left(u - \nu - \nu_S S\right)u_{cc}}{u_c^2} + \theta\right]e^{(r-\rho)(s-t_0)}ds +\\ &\quad +\Phi(\tau,t)\frac{\partial \psi^2(\tau)}{\partial c_0} > 0\end{aligned}$$

if $u(c(s)) \geq \nu(s) + \nu_S S(s)$ *for all* $s \in [t, \tau]$.

(iv) Complementarity to retirement age: $\frac{\partial \psi^2(t)}{\partial \tau} = 0$ *and*

$$\begin{aligned} \frac{\partial \psi^1(t)}{\partial \tau} &= \Phi(\tau^*, t)\left[-\frac{\nu(\tau^*, S(\tau^*)) + \nu_S S(\tau^*)}{u_c(c(\tau^*))} + (1-\theta)\, w(\tau^*)\right] \\ &= -\frac{\Phi(\tau^*, t)\left[\theta \nu(\tau^*, S(\tau^*)) + \nu_S S(\tau^*)\right]}{u_c(c(\tau^*))} \geq 0 \end{aligned} \tag{14}$$

if and only if $-\nu_S S(\tau^*) \geq \theta \nu(\tau^*, S(\tau^*))$.

(v) Annuity moral-hazard:

$$\frac{\partial \psi^i(t)}{\partial \theta} = A(t) > 0$$

if and only if $A(t) > 0$.

Ad (i) and (ii): Complementarity in health care over the life-cycle have been studied extensively (see e.g. Dow et al. 1999, Murphy and Topel 2006). By reducing the mortality from some disease 1, say, health care targeted at disease 1 contributes towards increasing the value of survival and, thereby, towards increasing the spending incentive for reducing mortality from another disease 2. Within our model complementarity is sequential: it relates either to health care purchased in the past (backward complementarity) or to health care to be purchased in the future (forward complementarity). The latter can be considered as nested, in the sense that both benefits and costs of future health care are included in the current VOH. Forward complementarity arises if and only if the annuity market is subject to (some) moral hazard ($\theta > 0$). This is because under moral hazard, future health expenditure is not comprised in the VOH, implying that only the marginal benefit $\mu_h\left(\widehat{t}\right)\psi^i(\widehat{t}) = 1 > 0$ is counted, where the equality holds under optimal future spending, as by (12). In contrast, in the first-best ($\theta = 0$) individuals will take full account of the expenditure at $\widehat{t}$, which under optimal future spending fully cancels with the marginal VOH, implying that future health care has no impact on the current VOH. Under backward complementarity, past expenditure and past gains to survival are 'sunk' and do not count towards the current VOH. Backward complementarity arises if and only if past health expenditure, by maintaining the current stock of health, contributes towards lower morbidity, i.e. a lower disutility from work, without reducing the value of current morbidity reductions by too much. In contrast to forward complementarity, backward complementarity is then effective even in a first-best.

Ad (iii): An increase in the consumption level always raises the post-retirement VOH and raises the pre-retirement VOH if the disutility of labour is sufficiently low as to generate a non-negative period utility $u(c(s)) - \nu(s, S(s)) \geq 0$ across the working life. As one would expect, health is complementary to consumption as a greater 'quality' of life raises the incentive to survive. This effect is even more pronounced in a second-best ($\theta > 0$), where future expenditure on consumption is not (or only partially) internalised in the VOH.

Ad (iv): While post-retirement health is unrelated to the retirement age, which by then is 'sunk', the impact of retirement on the pre-retirement VOH is ambiguous. Using the elasticity $\eta(\nu, S) := -\nu_S S \nu^{-1}$, pre-retirement health is complementary to the retirement age if and only if $\eta(\nu, S) \geq \theta$, i.e. if and only if the morbidity effect, as measured by the elasticity, is sufficiently strong. More specifically, this implies that (i) in a first-best ($\theta = 0$) health is always (weakly) complementary to retirement age; and (ii) in a second best ($\theta = 1$) health is substitutive to retirement age if the morbidity effect is weak (or absent), i.e. if $\eta(\nu, S) < 1$. First-best complementarity arises for the following reason. From the first-line RHS in (14) we note that an increase in the retirement age has three effects on the VOH: (a) it triggers an additional year's worth of disutility (a negative incentive); (b) it raises the incentive to lower morbidity as an additional working year needs to be accommodated (a positive incentive); (c) it generates an additional year's worth of wage income (a positive incentive). However, this last effect is only counted under a first-best ($\theta = 0$), where under an optimal retirement decision (13) it offsets the disutility of work. Hence, under a first-best, the gains from a morbidity reduction alone contribute towards (weakly) raising the VOH. In contrast, in a second-best ($\theta = 1$) the increase in wealth from a postponement of retirement remains unaccounted for, which leaves the offsetting effects (a) and (b). Health is then complementary to a postponement of retirement if and only if health improvements lead to sufficient reductions in the disutility of working. While this may be true for professions in which the disutility from working is strongly health-dependent, for occupations in which health does not bear strongly on the disutility, postponement of retirement may well lead to a reduction in pre-retirement health care.

Ad (v): Finally, we find that a switch from a first-best (individualised) annuity market to a second-best setting induces an increase in health care for all life-years in which the individual holds positive annuity wealth. This conforms precisely with the notion of longevity-related moral hazard (Davies and Kuhn 1992, Philipson and Becker 1998, Sheshinski 2008). We also note that if the individual is in debt, the moral-hazard incentive is reversed, leading to a reduction in health spending. In order to avoid ambiguity with regard to moral hazard, we assume for the remainder of the analytical section that $A(t) \geq 0$ for all $t \in [t_0, T]$, implying that moral hazard will always tend to induce over-investment in health.

We conclude this section by considering how the optimal retirement age depends on the level of consumption and the level of health expenditure. By straightforward differentiation of the matching condition (13) we obtain the following results.

Lemma 3 *(i) Complementarity of leisure to consumption:*

$$\frac{\partial \tau^*}{\partial c_0} = \frac{-w(\tau^*)u_{cc}e^{(r-\rho)(\tau^*-t_0)}}{w_t u_c(c^*(\tau^*)) + w(\tau^*)u_{cc}(r-\rho)c^*(\tau^*) - (\nu_t + \nu_S S_t)} < 0.$$

(ii) Complementarity of retirement-age to pre-retirement health:

$$\frac{\partial \tau^*}{\partial h(t)}\Big|_{t\in(t_0,\tau)} = \frac{-\nu_S S(\tau^*)\mu_h(t)}{w_t u_c(c^*(\tau^*)) + w(\tau^*)u_{cc}(r-\rho)c^*(\tau^*) - (\nu_t + \nu_S S_t)} \geq 0.$$

All else equal, the optimal retirement age decreases unambiguously in the level of consumption, which is reflecting that leisure (enjoyed during the retirement phase) is complementary to consumption. While post-retirement health care has no direct impact on the retirement age, pre-retirement health care raises the retirement age, whenever it lowers the disutility of labour, $\nu_S < 0$. Note, however, that whenever $\nu_S = 0$, health has no direct impact on the entry into retirement.

3 Allocational impact of moral hazard within the annuity market

We now examine the impact of moral hazard within the annuity market. Similar to Sheshinski (2008) and in contrast to Davies and Kuhn (1992) and Philipson and Becker (1998) we consider in addition the impact of moral hazard on retirement. We generalise the analysis in Sheshinski (2008) by considering (i) a whole pattern of health-investments over the life course, rather than a one-off investment at the beginning of life; and, more importantly, (ii) by allowing health not only to affect survival but also morbidity. As we will see, this latter aspect makes an important difference to the life-cycle effects of moral hazard. In order to facilitate the presentation, we consider the special case $\nu_S = 0$ first, before proceeding to the case $\nu_S < 0$. From now on we index by 'fb' and 'sb', respectively, first-best and second-best allocations, the latter involving moral hazard effects. The following Proposition is proved in Appendix C.

Proposition 1 *Suppose the absence of morbidity related effects, i.e.* $\nu_S = 0$. *Moral hazard in the annuity market then implies:* $h^{sb}(t) > h^{fb}(t)\, \forall t \in [t_0, T)$, $\tau^{sb} > \tau^{fb}$ *and* $c^{sb}(t) < c^{fb}(t)\, \forall t \in [t_0, T]$.[18]

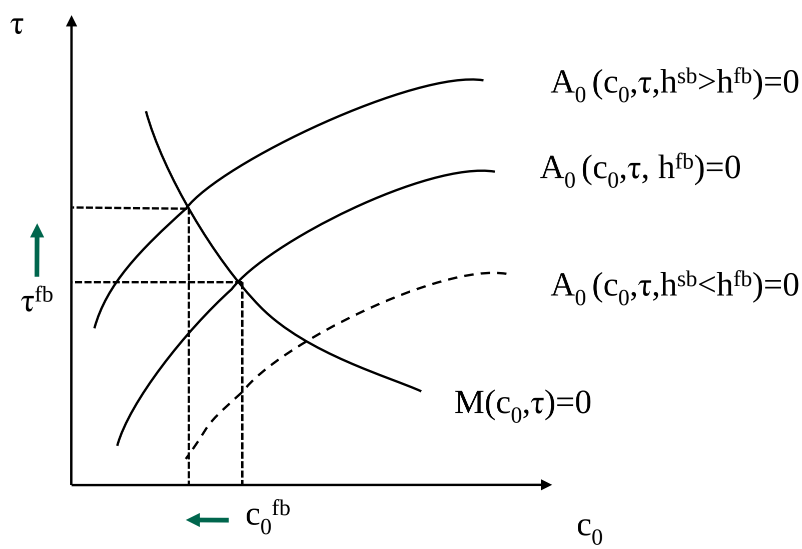

Figure 1: First-best vs. second-best with $\nu_S = 0$.

[18]For the final period, we obtain $h^{sb}(T) = h^{fb}(T) = 0$.

The proposition can be understood with reference to a graphical argument presented in Figure 1. The figure shows in (c_0, τ) space the downward-sloped optimality locus $M(c_0, \tau) := w(\tau) - \frac{\nu(\tau)}{u_c(c_0 e^{(r-\rho)(\tau - t_0)})} = 0$, depicting the combinations (c_0, τ) that satisfy the first-order condition for retirement (13), and the upward-sloped feasibility locus $A_0(c_0, \tau, h) := \int_{t_0}^{\tau^*} \Phi(s, t_0) w(s)\, ds - \int_{t_0}^{T} \Phi(s, t_0)(c(s) + h(s))\, ds = 0$, depicting the combinations (c_0, τ) that for a given schedule of health expenditure $\mathbf{h} := \{h(s) \,|s \in [t_0, T]\}$ lead to a balanced life-cycle budget. Recall from Lemma 3 that the optimal retirement age is strictly decreasing in consumption c_0 and independent of health care for $\nu_S = 0$. Budget feasibility implies that, for a given schedule of health expenditure $\mathbf{h}$, a greater level of consumption implies a higher retirement age. The optimal allocation for a given schedule of health expenditure is then found at the point of intersection of the 'optimality' and 'feasibility' locus. Thus, Figure 1 depicts the first-best allocation at the point, where $M\left(c_0^{fb}, \tau^{fb}\right) = A_0\left(c_0^{fb}, \tau^{fb}, \mathbf{h}^{fb}\right) = 0$. Recalling from part (i) of Lemma 2 the strict forward complementarity of health expenditure, a second-best can then involve two possible outcomes: $\mathbf{h}^{sb} \leq \mathbf{h}^{fb}$ or $\mathbf{h}^{sb} > \mathbf{h}^{fb}$, where the level of health expenditure is either non-increased along the whole schedule or increased along the whole schedule. We show in Appendix C that an increase (decrease) in the level of health expenditure along the schedule $\mathbf{h}$ implies an upward (downward) shift of the feasibility locus $A_0(c_0, \tau, \mathbf{h}) = 0$. A non-increasing level of health expenditure $\mathbf{h}^{sb} \leq \mathbf{h}^{fb}$ would thus imply $c_0^{sb} \geq c_0^{fb}$ and $\tau^{sb} \leq \tau^{fb}$. Using the complementarity relationships in parts (iii)-(v) of Lemma 2 we see that this generates a contradiction: If the level of consumption was to increase and retirement age was to decrease this would in itself call for an increase in health expenditure over the life-cycle. But even if consumption and retirement were to stay constant, then moral hazard alone would call for an increase in health expenditure. Thus, health expenditure must, indeed, increase $\mathbf{h}^{sb} > \mathbf{h}^{fb}$, which in turn implies that $c^{sb}(t) < c^{fb}(t)$ and $\tau^{sb} > \tau^{fb}$. The excess spending on health care and the need to accommodate an excessive length of life leads to a reduction in life-cycle consumption as in Davies and Kuhn (1992) and Philipson and Becker (1998). The individual partially compensates for this by postponing retirement as in Sheshinski (2008). Thus, excessive 'quantity' of life leads to a reduction in the 'quality' of life in terms of both foregone consumption and leisure.

We can contrast this outcome now for the case in which morbidity matters by affecting the disutility of labour, $\nu_S < 0$.

Proposition 2 *Suppose the presence of morbidity related effects, i.e.* $\nu_S < 0$*. Moral hazard in the annuity market then implies:* $h^{sb}(t) > h^{fb}(t) \, \forall t \in [t_0, T)$, $\tau^{sb} > \tau^{fb}$ and $c^{sb}(t) \geq c^{fb}(t)$ *if the reduction in morbidity is sufficiently strong.*

The argument follows in close analogy to the graphical argument for the special case $\nu_S = 0$ and can be developed with reference to Figure 2.

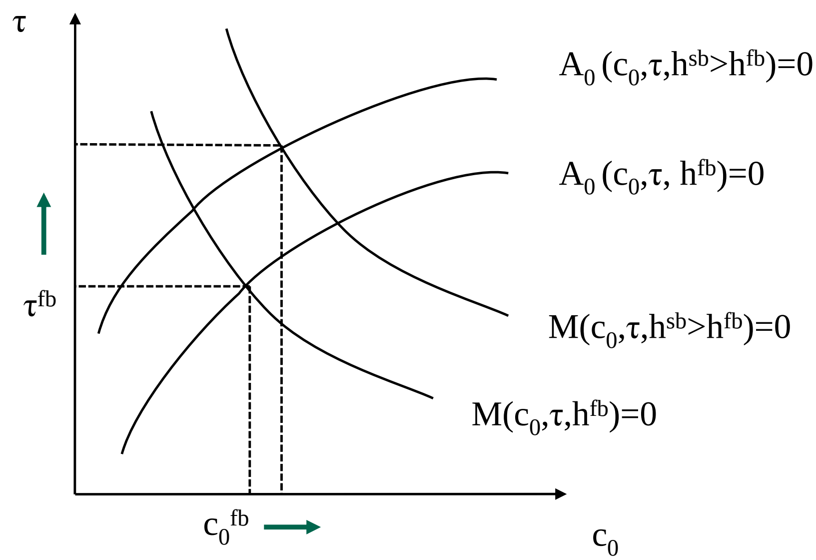

Figure 2: First-best vs. second-best with $\nu_S < 0$.

Similar to the previous case, the figure illustrates the feasibility locus $A_0(c_0, \tau, \mathbf{h}) = 0$ and optimality locus $M(c_0, \tau, \mathbf{h}) := w(\tau) - \frac{\nu(\tau, S(\tau))}{u_c(c_0 e^{(r-\rho)(\tau - t_0)})} = 0$, the only difference being that the optimality locus now also depends on the schedule of health expenditure $\mathbf{h}$. According to part (ii) of Lemma 3, retirement age is complementary to pre-retirement health so that in connection with forward and backward complementarity of health care (parts (i) and (ii) of Lemma 2) we have $\frac{\partial M(c_0, \tau, \mathbf{h})}{\partial \mathbf{h}} > 0$, implying that increases in $\mathbf{h}$ along the life-course imply an upward-shift of the optimality locus. With better pre-retirement health, it is optimal for the individual to retire later for any given level of consumption. But then we see that starting from the first-best allocation $M\left(c_0^{fb}, \tau^{fb}, \mathbf{h}^{fb}\right) = A_0\left(c_0^{fb}, \tau^{fb}, \mathbf{h}^{fb}\right) = 0$ an expansion of health expenditure due to moral hazard, $\mathbf{h}^{sb} > \mathbf{h}^{fb}$, leads to an upward shift of both optimality and feasibility locus. This necessarily implies an increase in the retirement age $\tau^{sb} > \tau^{fb}$, where the 'pull' for a later retirement arising from the reduction in mortality is now reinforced by a 'push' for later retirement arising from the reduction in morbidity. The impact on consumption is a priori ambiguous; however, as is readily seen from Figure 2 if the morbidity effect generates a sufficient outward-shift of the optimality locus, the increase in life-cycle income resulting from the large increase in retirement age allows for an increase in consumption over the first-best level, $c_0^{sb} > c_0^{fb}$. Thus, almost paradoxically the second-best outcome under moral hazard leads to a longer life (in better health), lower disutility from working and greater life-cycle consumption. Hence, what might appear as an improvement is nevertheless suboptimal, as now the individual is retiring 'much' too late.

In the following we render a more precise set of conditions under which moral hazard leads to an expansion in the consumption level, $c_0^{sb} \geq c_0^{fb}$. Assume that the disutility of labour is only dependent on the stock of health but not directly on age, such that $\nu(t, S) = \nu(S)$. We can then prove the following (see Appendix D).

Proposition 3 *(i) Generally,* $c_0^{sb} \geq c_0^{fb}$ *if and only if* $\frac{\nu\left(S\left(\tau^{sb},\mathbf{h}^{sb}\right)\right)}{\nu\left(S\left(\tau^{fb},\mathbf{h}^{fb}\right)\right)} \leq \frac{u_c\left(c_0^{fb} e^{(r-\rho)\left(\tau^{sb}-t_0\right)}\right)}{u_c\left(c_0^{fb} e^{(r-\rho)\left(\tau^{fb}-t_0\right)}\right)} \frac{w\left(\tau^{sb}\right)}{w\left(\tau^{fb}\right)}$.

(ii) If $r \leq \rho$ *and* $w\left(\tau^{sb}\right) \geq w\left(\tau^{fb}\right)$ *then* $c_0^{sb} \geq c_0^{fb}$ *if* $S\left(\tau^{sb}, \mathbf{h}^{sb}\right) \geq S\left(\tau^{fb}, \mathbf{h}^{fb}\right)$.

Generally, the second-best level of consumption is raised over and above the first-best level if the ratio between the disutility of labour at the point of retirement in the second-best as opposed to the first-best is sufficiently small in relation to the weighted ratio of the wages at the points of second-best as opposed to first-best retirement. This suggests that moral hazard tends to increase consumption levels if at the same time the stock of health at the point of second-best retirement is sufficiently large relative to the stock of health at the point of first-best retirement. This intuition is, indeed, confirmed for the case that $r \leq \rho$ implies a non-increasing stream of consumption and

$w\left(\tau^{sb}\right)\geq w\left(\tau^{fb}\right)$ implies a non-increasing stream of wages over the interval $\left[\tau^{fb},\tau^{sb}\right]$. In this case, it is sufficient that the stock of health at the point of second-best retirement does not fall short of the stock of health at the point of first-best retirement, $S\left(\tau^{sb},\mathbf{h}^{sb}\right)\geq S\left(\tau^{fb},\mathbf{h}^{fb}\right)$. Note that a priori, it is not clear whether the relationship holds: On the one hand, the second-best postponement of retirement, $\tau^{sb}>\tau^{fb}$, implies a degradation of health relative to the first-best; on the other hand, excessive second-best health investments, $\mathbf{h}^{sb}>\mathbf{h}^{fb}$, imply a greater stock of health. Considering small changes $\Delta\tau=\tau^{sb}-\tau^{fb}>0$ and $\Delta h(t)=h^{fb}(t)-h^{sb}(t)>0$ for $t\in[t_0,T]$, a greater stock of health at the point of second-best retirement obtains if and only if

$$\begin{aligned}
&-\mu\left(\tau^{fb},h^{fb}\right)S\left(\tau^{fb}\right)d\tau-\int_{t_0}^{\tau^{fb}}\mu_h\left(t,h\left(t\right)\right)S\left(\tau^{fb}\right)dh\left(t\right)\\
=&-\left[\mu\left(\tau^{fb},h^{fb}\right)\Delta\tau+\int_{t_0}^{\tau^{fb}}\mu_h\left(t,h\left(t\right)\right)\Delta h\left(t\right)dt\right]S\left(\tau^{fb}\right)\geq 0.
\end{aligned}$$

This holds if and only if $\mu\left(\tau^{fb},h^{fb}\right)\Delta\tau\leq-\int_{t_0}^{\tau^{fb}}\mu_h\left(t,h\left(t\right)\right)\Delta h\left(t\right)dt$,i.e. if and only if the aggregate reduction in the mortality rate due to additional pre-retirement health expenditure at least compensates for the mortality arising over the extended working-life. This intuition carries over to the case where wages are decreasing monotonously over the interval $\left[\tau^{fb},\tau^{sb}\right]$. Here, it is necessary for $c_0^{sb}\geq c_0^{fb}$ that $\nu\left(S\left(\tau^{sb},\mathbf{h}^{sb}\right)\right)<\nu\left(S\left(\tau^{fb},\mathbf{h}^{fb}\right)\right)$, implying that the second-best stock of health must be strictly greater than the first-best stock of health, i.e. $S\left(\tau^{sb},\mathbf{h}^{sb}\right)>S\left(\tau^{fb},\mathbf{h}^{fb}\right)$, and that the impact on the disutility of labour, as measured by the elasticity $\eta\left(\nu,S\right)$ is sufficiently pronounced. The same goes if consumption is increasing over the life-cycle for $r>\rho$.

Whether or not $S\left(\tau^{sb},\mathbf{h}^{sb}\right)>S\left(\tau^{fb},\mathbf{h}^{fb}\right)$ holds is difficult to gauge, not the least because it amounts to a counterfactual: While, in reality, we would expect a second-best situation with moral hazard in the annuity market it is not clear as to what would be the optimal level of survival at the point of retirement within a first-best allocation. Note, however, the analogy to the findings by Bloom et al. (2007) who consider the effects of a proportional compression of morbidity: for a health stock $S\left(t,z\right)$ that is declining with age, $S_t<0$, an increase in life-expectancy z by a factor ϕ comes with an increase in the stock of health such that $S(\phi t, \phi z) = S(t, z)$.[19] Thus, after an increase in life-expectancy by a factor $\phi = 1.1$ from 70 to 77 years, say, the current probability to reach an age of 66 years is the same as the probability to attain age 60 before the increase in life-expectancy. Bloom et al. (2007) then show that an increase in life expectancy by an amount $dz = (\phi - 1) z$ triggers a less than proportionate increase in the retirement age $d\tau < (\phi - 1)\tau$ as long as life-cycle consumption does not decrease. This in turn implies that the health stock at the point of deferred retirement must have increased: $S(\tau + d\tau, \phi z) > S(\phi\tau, \phi z) = S(\tau, z)$ since $\tau + d\tau < \phi\tau$.[20] Hence, although for rather different reasons, in their model, too, non-decreasing consumption goes together with a greater stock of health at the point of deferred retirement.

4 Policy Implications

In this section we consider two policy interventions aimed at eliminating or at least curtailing the moral hazard incentives arising within an imperfect annuity market with $\theta = 1$. We consider, in turn, an age-specific transfer scheme that fully offsets the moral hazard incentives and implements the first-best, and the scope for mandatory early retirement, as a second-best policy.

4.1 Optimal tax on health investments

Consider a transfer scheme, where an age t individual pays an age-specific tax $\alpha(t)$ on each unit $h(t)$ and receives a lump-sum transfer $\alpha(t) h^{fb}(t)$,

[19]The relationship $S_z(t, \cdot) > 0$ between health and life-expectancy z obviously refers to a dependency that is entirely different from the impact of health expenditure $S_h(t, \cdot) > 0$ considered by us.

[20]Consider the following example: Suppose life-expectancy increases by a factor $\phi = 1.1$ from $z = 70$ to $\phi z = 77$, so that $dz = 7$ years. Suppose that the retirement age increases from age $\tau = 60$ by 5 years to age 65. It follows that $d\tau = 5 < 6 = (\phi - 1)\tau$. Since $\tau + d\tau = 65 < 66 = \phi\tau$ it then follows that $S(65, 77) > S(66, 77) = S(60, 70)$. Survival probability and stock of health at the new age of retirement are higher than at the old retirement age.

implying a budget constraint

$$\begin{aligned}\dot{A}(t) &= w(t)+\alpha(t)\,h^{fb}(t)-c(t)-(1+\alpha(t))\,h(t)+(r+\overline{\mu}(t))A(t), \quad \text{for } t<\tau \\ \dot{A}(t) &= \alpha(t)\,h^{fb}(t)-c(t)-(1+\alpha(t))\,h(t)+(r+\overline{\mu}(t))A(t), \quad \text{for } t>\tau. \\ & A(t_0)=A(T)=0\end{aligned}$$

The individual's FOC for health care in the presence of moral hazard ($\theta = 1$) then runs

$$-\psi^i_{sb}(t)\mu_h(h^{sb}(t)) - \alpha(t) = 1,$$

where $\psi^i_{sb}(t) := \psi^i(t)\,|_{\theta=1}$. Combining this with the first-best FOC yields

$$\alpha(t) = \psi^i_{fb}(t)\mu_h(h^{fb}(t)) - \psi^i_{sb}(t)\mu_h(h^{sb}(t)),$$

where $\psi^i_{fb}(t) := \psi^i(t)\,|_{\theta=0}$. If the transfer induces a first-best pattern of health care $h^{sb}(t) = h^{fb}(t)$, then mortality rates, consumption levels and the retirement age correspond to the first-best as well. The fact that both consumption and retirement age approach their first-best levels follows from the complementarity relationship in part (i) of Lemma 3. We then obtain

$$\alpha^{fb}(t) = \left[\psi^i_{fb}(t) - \psi^i_{sb}(t)\right]\mu_h(h^{fb}(t)) = -A^{fb}(t)\,\mu_h(h^{fb}(t)).$$

Thus, the transfer corresponds exactly to the 'missing' impact of health investments - at the first-best level - on the annuity return. It therefore depends on the first-best level of wealth at age t weighted with the impact of health on mortality at this age. The transfer constitutes a tax if $A^{fb}(t) > 0$ and a subsidy otherwise. Note that the transfer can be implemented without the planner's knowledge of the actual level of health investments. However, in as far as it depends on the first-best level of health investments its implementation would be compromised in a world in which individuals are heterogeneous with regard to their demand for health care.

Using $-\mu_h(h^{fb}(t)) = \psi^i_{fb}(t)$, we can rewrite

$$\alpha^{fb}(t) = \frac{A^{fb}(t)}{\psi^i_{fb}(t)} = \frac{A^{fb}(t)}{\psi^i_{sb}(t) - A^{fb}(t)}$$

Thus, the transfer, obviously, increases in the individuals wealth and decreases in the 'gross' VOH, $\psi^i_{sb}(t)$, that is governing the choice of health care in the presence of moral hazard. By total differentiation we obtain

$$\frac{\dot{\alpha}^{fb}(t)}{\alpha^{fb}(t)} = \frac{\psi^i_{sb}(t)}{\psi^i_{fb}(t)}\left[\frac{\dot{A}^{fb}(t)}{A^{fb}(t)} - \frac{\dot{\psi}^i_{sb}(t)}{\psi^i_{sb}(t)}\right]$$

Thus, the transfer increases with the individual's age at a rate that is proportional to the growth rate of wealth net of the growth rate of the 'gross' VOH.

4.2 Mandatory retirement as a second-best policy

Provided that individuals are not in debt, longevity-related moral hazard implies the taxation of health expenditure at substantial rates (see Figure 6). Such a policy is not reflective of the real-world where health insurance arrangements imply the subsidisation of health care. Thus, if anything, real-world moral hazard problem tends to be exacerbated. In the light of this, a planner may use mandatory retirement as a second-best policy. Consider a small change in the retirement age away from it's (second-best) optimum τ^{sb}. The impact of this on the life-cycle loss from moral hazard, $V^{fb} - V^{sb}$, is then given by

$$\begin{aligned} \frac{d\left(V^{fb} - V^{sb}\right)}{d\tau}\Big|_{\tau=\tau^{sb}} &= -\int_{t_0}^{T} \mu_h(h^{sb}(t)) A\left(t\right) u_c\left(c^{sb}\left(t\right)\right) S\left(t\right) e^{-\rho(t-t_0)} \frac{dh^{sb}(t)}{d\tau} dt \\ &= u_c\left(c_0\right) \int_{t_0}^{T} \Phi\left(t, t_0\right) \frac{A\left(t\right)}{\psi_{sb}^{i}(t)} \frac{dh^{sb}(t)}{d\tau} dt, \qquad (15) \end{aligned}$$

where we have applied the envelope theorem. Here, $-u_c\left(c_0\right)\Phi\left(t, t_0\right)\frac{A(t)}{\psi_{sb}^{i}(t)} < 0$ for $A\left(t\right) > 0$ represents the expected marginal loss in life-cycle utility from over-investments in period t. Thus, a reduction in health expenditure within this period $dh^{sb}(t) < 0$ would lead to a reduction in the loss from moral hazard.[21] Continuing to assume $A\left(t\right) \geq 0$ for all t we then find that $\frac{dh^{sb}(t)}{d\tau} \geq 0$ is sufficient for a small reduction in the retirement age from τ^{sb} (e.g. in the form of mandatory early retirement) to yield a welfare improvement. Indeed, we are able to show that this is true without ambiguity for the case where the morbidity elasticity $\eta\left(v, S\right)$ is equal or otherwise close to one (see Appendix E for a proof).

Proposition 4 *If $\eta\left(\nu, S\right) = 1$ then a (small) reduction of the retirement age below τ^{sb} leads to an improvement in life-cycle utility.*

[21] Obviously, in periods in which the individual is in debt, we have $-u_c\left(c_0\right)\Phi\left(t, t_0\right)\frac{A(t)}{\psi_{sb}^{i}(t)} > 0$. In these periods the individual could reduce the loss on moral hazard by investing more.

The proposition establishes a case for a mandatory early retirement (at a limited scale) to curtail health-related moral hazard and, therefore, to contribute towards an improvement in the individual's life-cycle utility. Although the early retirement comes along with a reduction in consumption, it still benefits the individual. The reason is that the decrease in utility through both too early a retirement (given the level of health expenditure) and through a reduction in consumption are of second-order, whereas the increase in utility through the reduction in health expenditure is of first-order. The argument generalises to a setting, where early retirement incentives arise from actuarially unfair pension systems (Gruber and Wise 2004). Reversing the argument, the proposition suggests that increases in the statutory retirement age and/or incentives towards later retirement may reinforce longevity-related moral hazard. Indeed, they may trigger a reduction in life-time utility even if they bring with them improvements in health and a higher level of consumption.

Note that in many ways we would expect this result to carry over to the cases, where $\eta(\nu, S)$ differs from one, but the result is no longer as clear-cut. Consider for instance the special case where $\eta(\nu, S) = 0$. In this case, mandatory early retirement will still lead to a reduction in consumption and in post-retirement health care, the latter contributing to a reduction in moral hazard and, thus, to an improvement in social surplus. However, in this case, we cannot rule out that pre-retirement health care increases and, thereby, refuels moral hazard. Whether or not early retirement then still yields an increase in life-cycle utility depends on the relative size of the pre- and post-retirement effects.

5 Numerical analysis

In the following we apply numerical simulations to gain insight into the allocation of consumption, health care and retirement over the life-cycle, and more specifically into the inefficiencies generated by moral hazard within the annuity market. We model the mortality rate according to the proportional hazard model (see Kalbfleisch and Prentice 1980)

$$\mu(t, h(t)) = \tilde{\mu}(t)\phi(t, h(t)),$$

where $\tilde{\mu}(t)$ denotes the base mortality rate (effective in the absence of any health care) and $\phi(t, h(t))$ describes the impact of health care. While there is little evidence to guide our choice of the function $\phi(\cdot)$, it seems reasonable to assume the following properties: $\phi_h < 0, \phi_{hh} > 0$, $\phi_{ht} > 0$; $\phi(t, 0) = 1$ $(\forall t)$ and $\phi_h(t, 0) = -\infty$ $(\forall\, t)$. Thus, we specify

$$\phi(t, h(t)) = 1 - \sqrt{\frac{h(t)}{z}\frac{t-T}{1-T}} \tag{16}$$

with $z = 30$. The efficiency of health care is decreasing over age, and care becomes entirely ineffective for $t = T$, where we set the maximum life-span $T = 110$. Base mortality $\tilde{\mu}(t)$ is proxied by US-mortality for the years 1990-2000 taken from the Human Mortality Data Base (HMD). Per period utility is specified as

$$u(c(t)) = b + \frac{c(t)^{1-\sigma}}{1-\sigma}$$

where $b = 6$ and $\sigma = 1.5$. We assume that yearly earnings $w(t)$ are constant over the life-cycle up to the point of retirement. Furthermore, we assume $r = \rho = 0.06$. Setting the rate of time preference equal to the interest rate helps us to characterise the effects of the externalities on the age-profiles of the variables of interest. Specifically, for a constant consumption level over the life cycle the various age-dependent expressions relating to the VOL are not blurred by changes in the marginal utility of consumption over the life course.[22] With respect to the disutility of labour we distinguish two cases.

Case 1: The disutility depends on age but not on health, i.e. $\nu(t, S) \equiv \nu(t)$.

Case 2: A health-dependent disutility, for which we specify

$$\nu(t, S) = \nu(S) = \bar{z}(1 - S)$$

with $\bar{z} = 6.5$.[23] In order to make the two cases comparable to some degree[24] we assume that the health-independent disutility $\nu(t)$ in case 1 corresponds

[22]For this specification we obtain a statistical value of life for the age-group 35-39 that lies around \$3 million in the case of exogenous disutility of labour and around \$8 million in the case of endogenous disutility. Note that both figures lie in the range of typical empirical estimates as reported e.g. in Hall and Jones (2007).

[23]Note that for this specification we have $\nu_S = -\bar{z}$ and $\eta(\nu, S) = \frac{S}{1-S}$, implying that the elasticity decreases with age from a value $\eta(\nu, S)|_{t=t_0} = \infty$ to $\eta(\nu, S)|_{t=T} = 0$. We also find that $\eta(\nu, S) = 1 \Leftrightarrow S = 0.5$.

[24]We need to caution, however, that direct quantitative comparisons between the two cases are meaningless. This is because we are comparing across different sets of preferences/technologies as embraced by the different specifications of the disutility of labour.

to the health-dependent disutility in case 2 evaluated at the first-best pattern of survival $S_2^{fb}(t)$, $t \in [t_0, T]$ arising for case 2. Thus, $\nu(t) = \nu(S_2^{fb}(t))$, where $S_2^{fb}(t)$ is a constant for case 1.

We organise our comparison across six scenarios, three for each of the cases 1 and 2 with health-independent and health-dependent disutility of labour, respectively. These scenarios (summarised in Table 1) embrace the first-best, a setting with moral hazard (mh) and exogenous retirement (for simplicity at first-best level), and a setting with moral hazard and endogenous (second-best) retirement.

	Case 1: $\nu(t,S) \equiv \nu(t)$	Case 2: $\nu(t,S) = \nu(S)$
first-best ($\theta = 0; \tau = \tau^{fb}$)	scenario 1.1	scenario 2.1
mh with exog. retirement ($\theta = 1; \tau = \tau^{fb}$)	scenario 1.2	scenario 2.2
mh with endog. retirement ($\theta = 1; \tau = \tau^{sb}$)	scenario 1.3	scenario 2.3

Table 1: Scenarios

Figures 3 through 5 plot consumption, health investments and the value of health (VOH) for the six scenarios, where scenarios 1.1-1.3, corresponding to health-independent disutility, are depicted in the left hand panels and scenarios 2.1-2.3, corresponding to health-dependent disutility, are depicted in the right hand panels. The first-best scenarios 1.1. and 2.1 are depicted by a solid graph; moral hazard with exogenous retirement (scenarios 1.2 and 2.2) is depicted by a dashed graph; and moral hazard with endogenous retirement (scenarios 1.3 and 2.3) is depicted by a dotted graph. Since we assume $r = \rho$, all scenarios involve a constant stream of consumption across the life-course (i.e. perfect consumption smoothing) (see Figure 3). From Figure 4 we see that health care broadly follows a hump-shaped trajectory, reflecting that health investments are ineffective at young ages with very low base mortality and at the oldest ages with low returns, $\phi_h(\cdot)$, to health care.[25] We note, however, that in the case of health-dependent disutility of labour (the right hand panel of Figure 4) there is a local peak in expenditure

[25] A hump-shaped profile of health expenditure stands in contrast to the observation that in most countries (including the US) health expenditure strictly increases with age. The difference arises as our expenditure patterns follow the statistical or *ex-ante* VOL, which typically decreases from some age onwards (see e.g. the numerical exercises in Shepard

somewhat before the individual retires (the point of retirement corresponding to a local minimum). The demand for health care declines in the years leading up to retirement as, with a shortening of the remaining working life, health investments turn out less and less valuable for the purpose of lowering the disutility of labour. Indeed, we could interpret the decline in health investments with the advent of retirement as an anticipation effect. The downward-trend in health care is reversed at the point of retirement, where health investments begin to grow again as their effectiveness in curtailing mortality continues to increase with age up to the global peak. The change of investment incentives at the point of retirement is also reflected in the VOH (see Figure 5). While the VOH declines with age throughout for the case of health-dependent disutility of labour, the rate of decline is reduced at the point of retirement (right hand panel). This is because at this point the net value $-(\nu + \nu_S S)\, u_c^{-1}$, which for our specification is strictly positive, is fully written off.[26] In contrast, for the case with exogenous disutility of labour, the VOH reveals a local maximum at the point of retirement (left hand panel). The increase in the VOH over the period immediately preceding retirement reflects the anticipation that the disutility of labour $-\nu u_c^{-1} > 0$ will no longer depress the value of life from the point of retirement onward. In contrast to the case with health-dependent disutility, however, this does not lead to a reversal in the demand for health care which increases throughout, the main driver here being the increasing effectiveness of care over the relevant age range.[27]

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and Zeckhauser 1984, Murphy and Topel 2006). As Philipson et al (2010) argue, however, real health expenditure is driven by the *ex-post* VOL once a life threatening condition has materialised. At this point individuals are typically willing to spend a manifold of the ex-ante VOL. The bunching of life threatening situations at high ages then implies the increasing spending pattern. As our analysis is predominantly of a normative nature and not targeted at a realistic pattern of health expenditure per se, the discrepancy between our simulated expenditure and real-world expenditure is of minor consequence.

[26]Note that for our specification $-(\nu + \nu_S S) = -\overline{z}\,(1 - 2S) > 0$ since $S >> 0.5$ up to the age of retirement.

[27]In addition, we note that the VOH with health-dependent disutility of labour exceeds the VOH with health-independent disutility up to the point of retirement and then assumes a slightly lower level, reflecting the fact that due to earlier health investments the individual has maintained a larger stock of health. Similarly, it can be checked from Figure 4 that pre-retirement health investments tend to be considerably higher when they contribute towards reducing the disutility of labour. While this is intuitive, we caution against placing to much emphasis on these comparisons for the reasons outlined earlier.

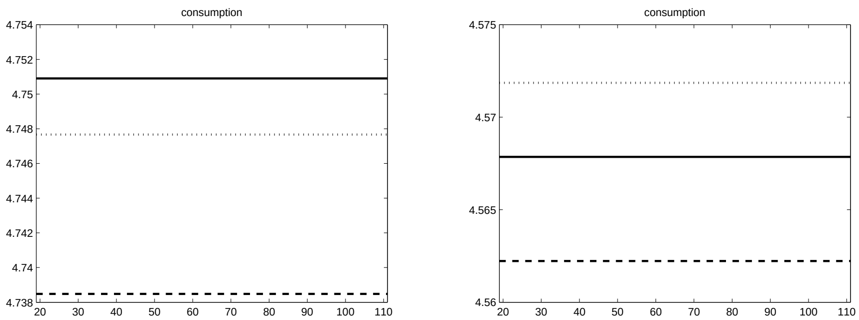

Figure 3: Consumption (left hand: case 1, right hand: case 2)

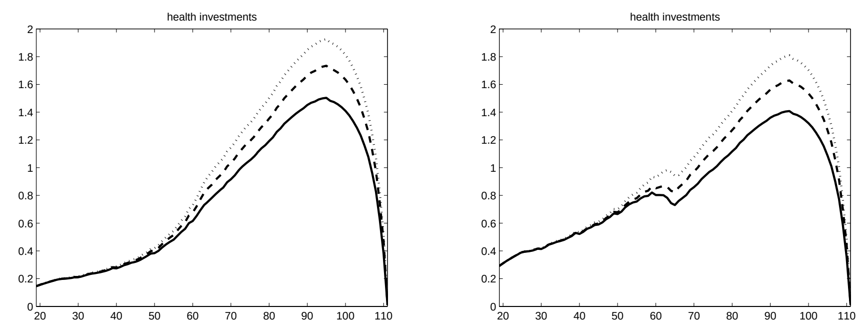

Figure 4: Health investments (left hand: case 1, right hand: case 2)

Having understood these general features of the allocation, we can now turn towards examining the impact of moral hazard. For the moment, we focus on a comparison of the scenarios 1.2 and 2.2 with moral hazard and *exogenous* retirement with the first-best scenarios 1.1 and 2.1. From Figures 3 and 4 we see that moral hazard (for a fixed retirement age) reduces the level of consumption below its first-best, while increasing the level of health expenditure. As expected, moral hazard also raises the value of health (see Figure 5).

When considering scenarios 1.3 and 2.3 with moral hazard and *endogenous* retirement we see that the postponement of retirement by about a year beyond the first-best level of 63.5 years and 63 years, respectively, allows the individual to increase both health expenditure and consumption relative

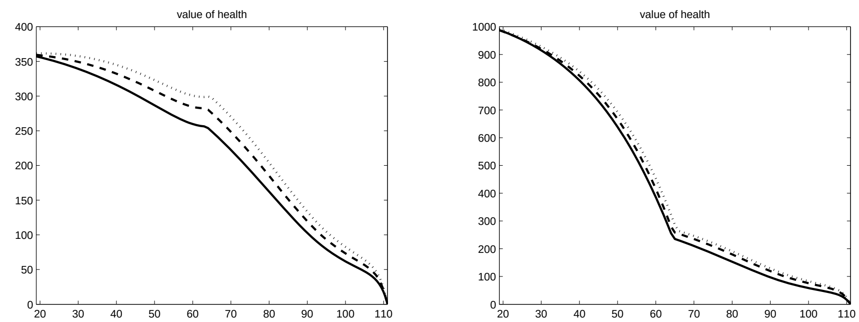

Figure 5: Value of health (left hand: case 1, right hand: case 2)

to scenarios 1.2 and 2.2 with exogenous retirement. The extension of the working life generates a higher life-cycle income which is used to accommodate both higher health investments and consumption. Indeed, all of this is true regardless of whether or not the disutility of labour is health dependent. The notable difference arises, however, when comparing the consumption levels under moral hazard with endogenous retirement (scenarios 1.3 and 2.3) to the first-best levels (scenarios 1.1 and 2.1). Here, we see from Figure 3 that while in case 1 with health-independent disutility (left hand panel) consumption continues to fall short of the first-best even after a postponement of retirement, this is no longer true in case 2 with health-dependent disutility (right hand panel). Indeed, in this case moral hazard increases both health expenditure and consumption, the latter even for an increase in the length of life. As we have argued before, this is nevertheless inefficient due to the excessive duration of working life. Finally, we note from Figure 4 that in the presence of health-dependent disutility, moral hazard leads to a significant increase in health investments (beyond the first-best) not only after retirement but also before its onset. Pre-retirement investments are raised in order to better accommodate the expansion of the working life. Again, this contrasts the case with health-independent disutility, where moral hazard leads to excessive health investments predominantly after retirement.

Figure 6 plots the age profile of the transfer that would implement a first-best allocation. For both cases 1 and 2, the transfer is sharply increasing over the life-cycle up to the point of retirement. Despite the decline in assets

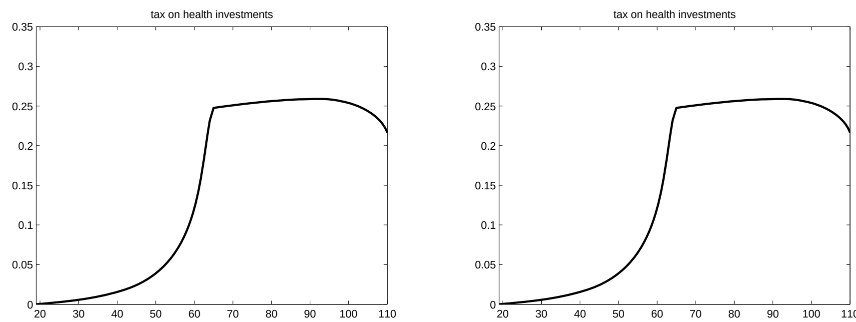

Figure 6: Transfer

from then on, the ongoing increase in the marginal productivity of health care continues to drive up the transfer by a modest amount before it declines towards the end of life. We conclude by noting that a first best would, indeed, imply the taxation of health care at a rate of about 25 per cent during the life years with peak spending. Undoubtedly such a policy would prove difficult to implement.

6 Conclusions

We have examined within a life-cycle model the nexus between health and retirement, paying close attention to the fact that both are determined endogenously and simultaneously. In contrast to previous work on health and retirement we take into account that health care contributes both to improved survival and longevity and to a reduction in morbidity, which in our model implies a lower disutility of work.

Our analysis shows that within a first-best world a greater demand for health and a higher age of retirement are complementary. The relationship is more ambiguous in the presence of moral hazard on the annuity market. In this case, an increase in the retirement age has a negative impact on the value of pre-retirement health care if the disutility of labour does not depend (much) on health. Here, the prospect of incurring a disutility of labour over additional life years lowers the value of life and, thus, the value of health spending. While this effect may be over-turned by the income effect arising from the increase in life-cycle income after a deferral of retirement, it leaves an ambiguity in the effect of retirement postponement on health care spending. Indeed, such a negative effect of an extended working life on health care spending is consistent with the evidence provided by Kerkhofs and Lindeboom (1997) and Lindeboom and Kerkhofs (2009). If, in contrast, the disutility of labour is sufficiently elastic in health then an increase in the retirement age leads to an additional incentive to spend on pre-retirement health. This case is consistent with the evidence provided by Dave et al. (2006), Kuhn, Wuellrich and Zweimüller (2010) and Behncke (2011) who show that (early) retirement has a negative impact on health.

A number of issues are worthy of note here. The empirical studies typically consider the effect of retirement on post-retirement health outcomes and then try to explain these by post-retirement changes in health-related behaviour (e.g. less exercise, less healthy nutrition, etc.). In contrast, our analysis shows that a significant part of the behavioural changes induced by a change in retirement age relate to the pre-retirement period and, therefore, to the anticipation of retirement. This is not inconsistent with the evidence in as far as pre-retirement behaviour determines the post-retirement stock of health. Indeed, it is not clear why changes in the retirement age should directly bear on post-retirement health incentives, which through the value of health are forwardlooking and therefore no longer contingent on the past event of retirement. Given that income effects are controlled for in the empirical studies, we should, indeed, expect that any remaining effects of retirement on health are governed by pre-retirement health behaviour.

We apply our analysis to study the impact of moral hazard within the annuity market when retirement matters and when it may depend on health not only through an income effect but also directly through the disutility of labour. Indeed, we show that the 'morbidity' impact of health on the disutility of labour may considerably alter the distortions arising through moral hazard. While both health investments and the length of the working life are always excessive relative to a first-best, moral hazard leads to an increase in consumption over and above the first-best level if the morbidity impact is strong. In this case the distortion towards (excessive) quantity of life and against the quality of life is no longer manifest in under-consumption, as it is according to the received literature, but only in an excessive working life. Consequently, early retirement policies, albeit distorting in many other ways, may have a role in contributing towards a reduction in health-related moral hazard. Conversely, pressures for an extension of the working life, while being justified on the grounds of sustainability of pension systems, should take into consideration the negative side-effect on health-related moral hazard.

While we believe our results to be insightful, they are obtained within a rather stylised framework and, therefore, require some comment regarding their limitations. First, our model lacks in as far as we assume morbidity to have only an impact on the disutility of labour but neither on earnings (productivity) nor directly on period utility. While the impact of health on the disutility of labour is justified by ample empirical evidence (e.g. Bound et al. 1999, Lindeboom and Kerkhofs 2009, Jones et al. 2010), it is equally clear that the other aspects of morbidity matter at least as much. Regarding the direct effect of health on utility, as captured e.g. in Ehrlich (2000) and Murphy and Topel (2006), we would argue that while it implies an additional incentive to invest in health over the life-course, this incentive relates both to the pre- and post-retirement phase of life and, therefore, has no qualitative bearing on the relationship between health and retirement over and above the mortality effect. Therefore, with the health and retirement nexus being the focus of the present analysis we deem it to be justified to omit the quality-of-life aspect of health. Regarding the effect of health on earnings a somewhat more diverse picture emerges. It could be argued that while a health-related reduction in the disutility of work typically pushes towards an increase in the retirement age, this may no longer be true if a wealth-effect from greater earnings becomes sufficiently strong. Furthermore, to the extent that earnings do not respond to changes in the individual's health care, this gives rise to an additional inefficiency in health spending: individuals would tend to spend too little when earnings are taken as given.

Second, a richer model would distinguish between morbidity and mortality related health care. In reality, many forms of health care tend to reduce both morbidity and mortality (e.g. the treatment of coronary heart disease or of chronic diseases), but certain relevant forms of care relate exclusively to morbidity (e.g. cataract surgery) or to mortality (e.g. treatment of acute myocardial infarction). As should be clear from our analysis such differentiated forms of health care interact distinctly - and quite possibly differently - with retirement incentives.

Finally, our framework disregards government intervention through retirement insurance. We should note, however, that our analysis does embrace the case of a fully-funded and actuarially fair pension scheme that gives rise to an efficient retirement incentive as long as the public pension does not fully crowd out private assets. If this is the case, then the individual is indifferent as to whether it accumulates assets in the form of private or public annuities and chooses the same life-cycle allocation. Furthermore, there is no reason to believe why public annuities should not be subject to the same moral hazard as private ones. The analysis of a pay-as-you-go pension scheme would require substantial additional modelling, in particular, as such an analysis would need to keep track of the interaction between the health care choices of different cohorts and, therefore, require an overlapping generations approach akin to the one pursued in Kuhn et al. (2011). We leave these extensions to future applications of our model.

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A Proof of Lemma 1

From the matching conditions and the necessary first order condition consumption and health expenditures are continuous at τ. The first order conditions also guarantee that both controls are continuous over the whole planning horizon. According to E1 the left hand side of the matching condition (5) is strictly lower than the right hand side at the beginning of the planning horizon. E2 implies the opposite at the end of the planning horizon. The continuity of both sides guarantees the existence of at least one $\tau \in (t_0, T)$ where both sides are equal.

U1 implies that consumption is non-decreasing over time. Thus marginal utility $u_c(c)$ does not increase over time. The disutility of work increases over time as the survival probability is strictly decreasing according to (1). Thus the left hand side of the matching condition is increasing and convex over time. U2 implies that the wage (representing the right hand side of the matching condition) is concave over time. Putting this together the solution τ has to be unique.[28] $\square$

B Proof of Lemma 2

Part (i): For $\widehat{t} \in (t, T)$ we write the post-retirement VOH as

$$\psi^2(t) = \int_t^{\widehat{t}} \Phi(s,t) \left\{ \frac{u\left(c\left(s\right)\right)}{u_c\left(c\left(s\right)\right)} - (1-\theta)\left[c\left(s\right) + h\left(s\right)\right] \right\} ds + \Phi(\widehat{t}, t)\psi^2(\widehat{t}),$$

and obtain

$$\frac{\partial \psi^2(t)}{\partial h\left(\widehat{t}\right)} = -\Phi(\widehat{t}, t)\left[\mu_h\left(\widehat{t}\right)\psi^2(\widehat{t}) + 1 - \theta\right] = \theta\Phi(\widehat{t}, t),$$

where the second equality follows from the FOC for $h\left(\widehat{t}\right)$ as in (12). For $\widehat{t} \in (\tau, T)$ we write the pre-retirement VOH as

$$\begin{aligned} \psi^1(t) &= \int_t^{\tau} \Phi(s,t) \Big(\frac{u\left(c\left(s\right)\right) - \left[\nu\left(s, S\left(s\right)\right) + \nu_S S\left(s\right)\right]}{u_c\left(c\left(s\right)\right)} + \\ &\quad (1-\theta)\left[w\left(s\right) - c\left(s\right) - h\left(s\right)\right] \Big) ds + \Phi(\tau, t)\psi^2(\tau). \end{aligned} \quad (17)$$

[28] We stress that the conditions are sufficient and not necessary. In particular, this applies to U1, a condition that is overly restrictive.

and obtain $\frac{\partial\psi^1(t)}{\partial h(\widehat{t})} = \Phi(\tau,t)\frac{\partial\psi^2(t)}{\partial h(\widehat{t})} = \theta\Phi(\widehat{t},\tau)\Phi(\tau,t) = \theta\Phi(\widehat{t},t)$. For $\widehat{t} \in (t,\tau)$ we write

$$\begin{aligned}\psi^1(t) &= \int_t^{\widehat{t}} \Phi(s,t)\Big(\frac{u(c(s)) - [\nu(s,S(s)) + \nu_S S(s)]}{u_c(c(s))} + \\ &(1-\theta)[w(s) - c(s) - h(s)]\Big)ds + \Phi(\widehat{t},t)\psi^1(\widehat{t})\end{aligned}$$

and obtain again $\frac{\partial\psi^1(t)}{\partial h(\widehat{t})} = \theta\Phi(\widehat{t},t)$.

Part (ii): For $\widehat{t} \in (t_0,t)$ we obtain in a straightforward way the expressions reported in the Lemma.

Parts (iii) and (iv): Using (8) and (17) we obtain the reported expressions from straightforward differentiation and cancellation.

Part (v): Follows from straightforward differentiation of (8) and (7) when observing that $A(t) = -[H(t) + E(t)]$.□

C Proof of Proposition 1

As a preliminary, consider the life-cycle budget

$$A(t_0) = \int_{t_0}^{\tau} \Phi(s,t_0)w(s)\,ds - \int_{t_0}^{T} \Phi(s,t_0)(c(s) + h(s))\,ds,$$

from which we obtain in a straightforward way $\frac{\partial A(t_0)}{\partial c_0} < 0$ and $\frac{\partial A(t_0)}{\partial \tau} > 0$ as well as

$$\begin{aligned}\frac{\partial A(t_0)}{\partial h(t)}\Big|_{t\in[t_0,\tau]} &= -\Phi(t,t_0)\left\langle \mu_h(t)\left\{\int_t^{\tau} w(s)\,ds - \int_t^{T}[c(s) + h(s)]\,ds\right\} + 1\right\rangle \\ &= -\Phi(t,t_0)\mu_h(t)\left[H(t) - E(t) - \psi^1(t)\right] \\ &= \Phi(t,t_0)\mu_h(t)\left[A(t) + \psi^1(t)\right] < 0\end{aligned}$$

and, similarly,

$$\begin{aligned}\frac{\partial A(t_0)}{\partial h(t)}\Big|_{t\in[\tau,T]} &= -\Phi(t,t_0)\left\langle -\mu_h(t)\int_t^{T}[c(s) + h(s)]\,ds + 1\right\rangle \\ &= -\Phi(t,t_0)\mu_h(t)\left[-E(t) - \psi^1(t)\right] \\ &= \Phi(t,t_0)\mu_h(t)\left[A(t) + \psi^1(t)\right] < 0\end{aligned}$$

when continuing to assume $A(t) > 0$. Life-cycle budget balance $A(t_0) = 0$ then implies

$$\frac{\partial A(t_0)}{\partial \tau} d\tau + \frac{\partial A(t_0)}{\partial c_0} dc_0 + \int_{t_0}^{T} \frac{\partial A(t_0)}{\partial h(t)} dh(t)\, dt = 0. \tag{18}$$

From parts (i) and (ii) of Lemma 3 we note that $\theta = 1$ must lead to either of two possible allocations (i): $c^{sb}(t) \geq c^{fb}(t)$ and $\tau^{sb} \leq \tau^{fb}$ or (ii) $c^{sb}(t) < c^{fb}(t)$ and $\tau^{sb} > \tau^{fb}$. In the following we show by contradiction that (i) cannot be a (second-best) optimum before shwoing that an allocation with $c^*(t) < c^{fb}(t)$, $\tau^* > \tau^{fb}$ and $h^*(t) > h^{fb}(t)\,\forall t \in [t_0, T)$ is, indeed, an optimum.

Thus, suppose $c^{sb}(t) \geq c^{fb}(t)$ and $\tau^{sb} \leq \tau^{fb}$ as part of an optimum with $\theta = 1$. Then parts (iii)-(v) of Lemma 2 together with backward complementarity in part (i) of Lemma 2 imply that $h^*(t) > h^{fb}(t)\,\forall t \in [t_0, T)$ must be true. However, it is now readily checked that such an allocation contradicts the life-cycle budget balance (18) and can, therefore, not be part of an optimum.

Thus, a second-best optimum at $\theta = 1$ must involve $c^{sb}(t) < c^{fb}(t)$ and $\tau^{sb} > \tau^{fb}$. From the life-cycle budget balance (18) and backward complementarity according to part (i) of Lemma 2 we then obtain $h^{sb}(t) > h^{fb}(t)$ for all $t \in [t_0, T)$. We conclude by noting that this is, indeed, compatible with the complementarity relationships described in Lemma 2: although parts (iii) and (iv) of Lemma 2 would call for $h(t) < h^{fb}(t)$, part (v) calls for $h^{sb}(t) > h^{fb}(t)$.□

D Proof of Proposition 3

Part (i): Using the matching condition (13), we find that $c_0^{sb} \geq c_0^{fb} \Leftrightarrow \frac{\nu\left(S\left(\tau^{sb}, \mathbf{h}^{sb}\right)\right)}{u_c\left(c_0^{fb} e^{(r-\rho)(\tau^{sb}-t_0)}\right)} \leq w\left(\tau^{sb}\right)$. Multiplying the RHS of the inequality with $\frac{\nu\left(S\left(\tau^{fb}, \mathbf{h}^{fb}\right)\right)}{w\left(\tau^{fb}\right) u_c\left(c_0^{fb} e^{(r-\rho)\left(\tau^{fb}-t_0\right)}\right)} = 1$ and rearranging gives the condition reported in the Proposition. Part (ii): Note that $r \leq \rho$ and $\tau^{sb} > \tau^{fb}$ imply $c_0^{fb} e^{(r-\rho)\left(\tau^{sb}-t_0\right)} \leq c_0^{fb} e^{(r-\rho)\left(\tau^{fb}-t_0\right)}$ and, therefore, $\frac{u_c\left(c_0^{fb} e^{(r-\rho)\left(\tau^{sb}-t_0\right)}\right)}{u_c\left(c_0^{fb} e^{(r-\rho)\left(\tau^{fb}-t_0\right)}\right)} \geq 1$. For $w\left(\tau^{sb}\right) \geq w\left(\tau^{fb}\right)$

the sufficiency condition then follows immediately when noting that $\nu_S < 0$. □

E Proof of Proposition 4

We need to distinguish

$$\frac{dh^{sb}(t)}{d\tau}|_{t<\tau} = \overbrace{\frac{\partial h^{sb}(t)}{\partial \tau}|t<\tau}^{\lesseqgtr 0 \text{ by Lemma 2(iv)}} + \overbrace{\frac{\partial h^{sb}(t)}{\partial c_0}}^{>0 \text{ by Lemma 2 (iii)}} \frac{dc_0}{d\tau} + \int_{t_0}^{T} \overbrace{\frac{\partial h^{sb}(t)}{\partial h\left(\widehat{t}\right)}}^{\geq 0 \text{ by Lemma 2 (i)\&(ii)}} \frac{dh^{sb}\left(\widehat{t}\right)}{d\tau} d\widehat{t},$$

$$\frac{dh^{sb}(t)}{d\tau}|_{t\geq\tau} = \overbrace{\frac{\partial h^{sb}(t)}{\partial c_0}}^{>0 \text{ by Lemma 2 (iii)}} \frac{dc_0}{d\tau} + \int_{t}^{T} \overbrace{\frac{\partial h^{sb}(t)}{\partial h\left(\widehat{t}\right)}}^{>0 \text{ by Lemma 2 (i)}} \frac{dh^{sb}\left(\widehat{t}\right)}{d\tau} d\widehat{t}.$$

Thus, the impact of retirement on pre-retirement health is composed of three effects: (i) the direct impact of retirement, which is ambiguous (Lemma 2 (iv)); (ii) the impact through a change in consumption which is positive if and only if $\frac{dc_0}{d\tau} > 0$ (observing Lemma 2 (iii)); (iii) the impact through complementary changes in health care across the life-cycle, which again is positive if $\frac{dh^{sb}(\widehat{t})}{d\tau} > 0$ (observing Lemma 2 (i) & (ii)). Here, we note that in the absence of a morbidity effect, $\frac{\partial h^{sb}(t)}{\partial h(\widehat{t})} = 0$ for $\widehat{t} \in [t_0, t]$. The impact of retirement on post-retirement health is composed of two effects: (iv) the impact through a change in consumption; and (v) the impact through backward complementarity. In principle, we can now distinguish three cases, depending on whether $\frac{\partial h^{sb}(t)}{\partial \tau}|_{t<\tau} = 0$, $\frac{\partial h^{sb}(t)}{\partial \tau}|_{t<\tau} < 0$ or $\frac{\partial h^{sb}(t)}{\partial \tau}|_{t<\tau} > 0$. For the purpose of this proof we focus on the case, where $\frac{\partial h^{sb}(t)}{\partial \tau}|_{t<\tau} = 0$, which ist true at $\tau = \tau^{sb}$ if $\eta(\nu, S) = 1$. We now show that for this case we obtain $\frac{dc_0}{d\tau} > 0$ and $\frac{dh^{sb}(t)}{d\tau} > 0$ for all $t \in [t_0, T)$ and, therefore, $\frac{dh^{sb}(t)}{d\tau} > 0$ for all $t \in [t_0, T)$.

First, we show that $sgn\frac{dh^{sb}(t)}{d\tau} = sgn\frac{dc_0}{d\tau}$ holds for $\forall t \in [t_0, T]$. Within this part of the proof we assume that $h^{sb}(t)$ is continuous with respect to τ for

$\forall t \in [t_0, T]$. As follows we show the relation for the case $t \geq \tau$. The opposite case $t < \tau$ is completely analogous. For $t = T$ we have

$$\frac{dh^{sb}(T)}{d\tau} = \frac{\partial h^{sb}(T)}{\partial c_0}\frac{dc_0}{d\tau} \tag{19}$$

The first fraction is positive due to Lemma 2, implying that the assertion $sgn\frac{dh^{sb}(t)}{d\tau} = sgn\frac{dc_0}{d\tau}$ holds for $t = T$. Because of continuity there exists an $\epsilon > 0$ such that the sign of $\frac{dh^{sb}(t)}{d\tau}$ does not change in $t \in [T-\epsilon, T]$. Consequently the assertion holds for the interval $[T-\epsilon, T]$. For arbitrary ϵ_2 we can write

$$\begin{aligned}\frac{dh^{sb}(T-\epsilon_2)}{d\tau} &= \frac{\partial h^{sb}(T-\epsilon_2)}{\partial c_0}\frac{dc_0}{d\tau} + \int_{T-\epsilon_2}^{T} \frac{\partial h^{sb}(T-\epsilon_2)}{\partial h(s)}\frac{dh^{sb}(s)}{d\tau}\, ds \\ &= \frac{\partial h^{sb}(T-\epsilon_2)}{\partial c_0}\frac{dc_0}{d\tau} + \int_{T-\epsilon}^{T} \frac{\partial h^{sb}(T-\epsilon_2)}{\partial h(s)}\frac{dh^{sb}(s)}{d\tau}\, ds + \\ &\quad + \int_{T-\epsilon_2}^{T-\epsilon} \frac{\partial h^{sb}(T-\epsilon_2)}{\partial h(s)}\frac{dh^{sb}(s)}{d\tau}\, ds \end{aligned} \tag{20}$$

The first term and the first integral are positive because of Lemma 2 and continuity. Because of continuity there exists an $\epsilon_2 > \epsilon$, such that $\frac{dh^{sb}(s)}{d\tau}$ does not change the sign in the interval $[T-\epsilon_2, T-\epsilon]$. Therefore the assertion holds for the interval $t \in [T-\epsilon_2, T]$. Applying this step iteratively we conclude that the assertion holds for $[t_0, T]$.

Second, we show that $\frac{dc_0}{d\tau} > 0$ by contradiction. Thus, suppose $\frac{dc_0}{d\tau} \leq 0$. From budget-balance (18) we would then obtain $\frac{dh^{sb}(t)}{d\tau} > 0$ at least for some t. However, this contradicts $sgn\frac{dh^{sb}(t)}{d\tau} = sgn\frac{dc_0}{d\tau}$. Therefore, $\frac{dc_0}{d\tau} > 0$ and $\frac{dh^{sb}(t)}{d\tau} > 0$ for all $t \in [t_0, T)$, which is compatible with (18) and, thus, the unique outcome. It then follows that the derivative in (15) is unambiguously positive, implying that a small reduction in τ below τ^{sb} leads to a welfare improvement.□

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