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Public education and economic prosperity: semi-endogenous growth revisited

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Abstract

We introduce publicly funded education into R&D-based economic growth theory. Our framework allows us to i) explicitly describe a realistic process of human capital accumulation within these types of growth models, ii) reconcile semi-endogenous growth theory with the empirical evidence on the relationship between economic development and population growth, and iii) revise the policy invariance result of semi-endogenous growth frameworks. In particular, we show that the model supports a negative association between economic growth and population growth if the education sector is well developed and the population growth rate is low, that is, for modern industrialized countries. Furthermore, within our framework, changes in public educational investments have the potential to affect the long-run balanced growth rate.

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Keywords: public education, human capital accumulation, technological change, semi-endogenous economic growth

1 Introduction

Over the last decades the role of education in the process of economic development has been analyzed extensively. Most empirical studies find a positive association between economic growth and measures for overall educational attainment (see for example Barro, 1991; Sachs and Warner, 1995; Bils and Klenow, 2000; Krueger and Lindahl, 2001)[1] and Lutz et al. (2008) even conclude that

> "...better education does not only lead to higher individual income but also is a necessary (although not always sufficient) precondition for long-term economic growth... Education is a long-term investment associated with near-term costs, but, in the long run, it is one of the best investments societies can make in their futures." (Lutz et al., 2008, p. 1048).

Despite these empirical findings and the seminal theoretical contributions of Lucas (1988), Galor and Weil (2000) and Cervellati and Sunde (2005) — showing different mechanisms by which education exerts a positive influence on economic prosperity — the main focus of research and development (R&D)-based growth theory has long been on technological progress as being determined by the R&D effort of an *uneducated* workforce. In one of the first models of this type, Romer (1990) acknowledges that the aggregate human capital stock of an economy and not raw, uneducated, labor is the driving force behind technological progress, but he does not model this idea explicitly. To put it differently, within these frameworks, the aggregate human capital stock exhibits the same behavior as raw labor and investments in education cannot be addressed. However, to underscore the vast importance of changes in education over the last decades, Table 1 shows the mean years of schooling of the population aged 15+ for the years 1960 and 2010 in the G-8 countries. There has been a huge increase over time, with annual growth rates between 0.5% and 2%. The table also displays pupil-teacher ratios in primary education, where the substantial decline over the corresponding time horizon indicates that educational investments per child and per year have also been rising.

Another disadvantage of early R&D-based growth models in the vein of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt

[1] However, the significance of this association and the direction of causality are often debated (cf. Durlauf et al., 2005).

Table 1: Mean years of schooling and pupil-teacher ratios in primary education for the G-8 countries

	Mean years of schooling		Pupil-teacher ratios	
Country	1960	2010	1971	2009
Canada	8.31	11.37	23.00*	17.42*
France	4.20	10.53	22.79	18.73
Germany	5.15	11.82	17.46*	13.00
Italy	4.86	9.88	21.62	10.33*
Japan	8.02	11.59	26.39	18.05
Russia	5.16	8.84	27.95*	18.06
U.K.	7.04	9.75	24.86*	17.96
USA	9.25	12.20	14.05	13.87

The data has been obtained from Barro and Lee (2010) and the World Bank (2012) "Education Statistics". Note that the indicated year differs for the entries marked with an asterisk because of missing data. The base years for pupil-teacher ratios are 1972 for Canada, 1995 for Germany, 1981 for Russia and 1985 for the USA. The end years for the same data series are 2000 for Canada and 2007 for Italy.

(1992) has been their support of a strong scale effect in the sense that the size of a countries' population determines its long-run economic growth prospects. The intuitive explanation is that larger populations feature i) larger markets and therefore more profit opportunities for innovative firms that introduce new products, and ii) a larger pool of labor, that is, more potential researchers available for R&D to propel technological progress. While it has been shown by Kremer (1993) that the scale effect was indeed important in economic history prior to the twentieth century for the world as a whole, it has been refuted by Jones (1995a) and Jones (1995b) for individual countries and their growth experiences in the second half of the twentieth century. This paved the way for semi-endogenous growth models (cf. Jones, 1995a; Kortum, 1997; Segerström, 1998) that remove the strong scale effect in a way that the long-run economic growth rate positively depends on population *growth* but not its *size*. The basic intuitive argument runs as follows: keeping up technological progress at an exponential rate becomes more and more difficult as the technological frontier expands. A constant inflow of scientists into the R&D sector is thus required to counterbalance this negative effect. In the long run, such a constant inflow can only be sustained by having positive population growth. However, even this implication has been severely criticized on the basis of empirical evidence that rather supports a *negative* association between economic growth and population growth (see for example Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Bernanke and Gürkaynak, 2001). Furthermore, the removal of the strong scale effect came at the price that the long-run economic growth rate within semi-endogenous growth models was rendered invariant to economic policy (see Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999, for potential ways to circumvent this implication).

Some recent attempts have been made to reconcile theory and evidence on the interrelation between population growth and economic growth. Dalgaard and Kreiner (2001), Strulik (2005) and Strulik et al. (2011) implement privately financed education into R&D-based growth models. While Dalgaard and Kreiner (2001) and Strulik (2005) emphasize that newborns do not have any education and therefore a larger birth rate slows down growth of average human capital and therefore hampers economic development, Strulik et al. (2011) rely on a child quality-quantity trade-off in the vein of Becker (1993) to show that a shift toward having fewer but better educated children can lead to a larger aggregate human capital stock and therefore to faster economic growth.

Some aspects of human capital accumulation in the context of R&D-based growth theory have also been analyzed in the articles by Eicher (1996), Arnold (1998), Funke and Strulik (2000) and Arnold (2002). In these contributions, however, the growth rate of the population is assumed to be zero. This means that human capital accumulation fully adopts the role that population growth has had in standard R&D-based semi-endogenous growth models and the effect of population growth and human capital accumulation cannot be addressed.

The aim of our paper is threefold. First, we want to implement the notion of *publicly* financed education into R&D-based economic growth frameworks. While the assumption of *privately* financed education might be justifiable for the United States, it does not fit for European countries because there, education systems are largely financed by the state (cf. Docampo, 2007; OECD, 2011, p. 232). Furthermore, we want to introduce a realistic production process for human capital by relaxing the implicit assumption often made in the literature that the sole input in human capital accumulation is time and effort by those to be educated (or by their parents). By contrast, our model features an education sector that employs teachers to build up the human capital stock of the next generation. Consequently, an increase in educational investments has the realistic side effect that labor is drawn away from other productive sectors of the economy.

Second, we attempt to reconcile theory and evidence by showing that our model allows for both a negative and a positive relationship between economic growth and population growth. The negative relationship is more likely to prevail for countries in which the education sector is well developed and population growth is slow, that is, typically for modern industrialized countries. This implication is consistent with the empirical findings of Brander and Dowrick (1994), Kelley and Schmidt (1995), Ahituv (2001) and Bernanke and Gürkaynak (2001). The positive relationship, on the other hand, is more likely to prevail for countries in which the education sector is badly developed and population growth is fast. Since this primarily applies for countries in an early stage of development, our results are also consistent with the empirical findings of Kremer (1993). However, we do not want to overstretch the R&D-based growth framework and acknowledge that it is only suitable for developed countries.

Third, we aim to reintroduce scope for policymakers to influence the long-run economic growth rate and show that public expenditures for education are crucial in this regard. This addresses a major concern of Dinopoulos and Thompson (1998) Peretto (1998), Young (1998) and Howitt (1999), who suggested that there is indeed room for policymakers to intervene with respect to long-run economic development. Furthermore, our result is consistent with the vast empirical literature on the interrelation between education and economic prosperity (cf. Barro, 1991; Sachs and Warner, 1995; Bils and Klenow, 2000; Krueger and Lindahl, 2001; Lutz et al., 2008).

The basic mechanism of our model is the following. Human capital is used as an input in three sectors that compete for it on the labor market: workers produce goods in the final goods sector, scientists produce ideas in the R&D sector and teachers produce human capital for the next generation in the education sector. The government collects taxes and uses the proceeds to pay the wages of the teachers. Consequently, an increase in taxes raises the number of teachers and thereby draws labor from the other sectors. This harms economic growth in the *short*- to *medium* run. However, the increase in the number of teachers fosters human capital accumulation and thereby increases productivity of the next generations. This in turn raises the *long*-run growth perspectives of the economy.

Some aspects of publicly financed education have been treated in the contributions of Blankenau and Simpson (2004) and Grossmann (2007). These papers are different to our approach because they do not consider an explicit education production sector that relies on teachers as input and they abstract from population growth. The analysis of Grossmann (2007) relies on R&D-based growth theory and shows that public education (in the form of subsidies for private education) is growth promoting. By contrast, Blankenau and Simpson (2004) use a model without R&D, where growth depends on human capital accumulation only. In their model public education has a direct positive impact on economic growth but can have negative general equilibrium effects via a negative impact on the physical capital stock and via crowding out of private education.

This paper proceeds as follows: Section 2 contains the theoretical model and the derivation of the growth rates of endogenous variables along the balanced growth path. We analytically assess the dependence of these growth rates on the underlying parameters, in particular, population growth and public educational expenditures. In Section 3 we numerically analyze the implications of an increase in public educational expenditures for economic growth during the transition to the new balanced growth path. Finally, Section 4 discusses the results, draws conclusions for economic policy and highlights scope for further research.

2 The model

This section describes the discrete time overlapping generations version of the R&D-based economic growth framework based upon Romer (1990) and Jones (1995a). Furthermore, we introduce a governmentally funded education sector and analyze its implications for long-run economic growth perspectives.

2.1 Basic assumptions

The demographic structure of our model economy follows Diamond (1965) and is a simplified version of Strulik et al. (2011). There are three phases of an individual's life cycle, each lasting for 25 years: childhood, adulthood and retirement. Children do not face economic decisions but they receive publicly funded education which determines their human capital level as an adult. Adults, whose cohort size at time t is given by L_t, inelastically supply their skills on the labor market, consume and save for retirement. The retirees in turn finance their consumption expenditures out of savings carried over from adulthood. We treat population growth as exogenous and assume that adults give birth to $n > 1$ children such that the population grows at rate $n-1$. Endogenizing population growth and private educational investments would severely complicate the model structure and obscure the basic mechanisms we aim to highlight. Therefore, we leave these issues for further research.

There are four sectors: final goods production, intermediate goods production, R&D and education. Two production factors can be used in these sectors: capital and labor. The latter is available in three different forms: i) workers in the final goods sector denoted by $L_{t,Y}$, ii) scientists in the R&D sector denoted by $L_{t,A}$, and iii) teachers in the education sector denoted by $L_{t,E}$. The final goods sector employs workers and machines supplied by the intermediate goods sector to produce for a perfectly competitive consumption good market. The Dixit and Stiglitz (1977) monopolistically competitive intermediate goods sector produces the machines for the final goods sector using capital as variable production factor and one machine-specific blueprint as fixed input. These blueprints are in turn supplied by the R&D sector which employs scientists to produce them. Finally, the education sector employs teachers to produce individual human capital for the next generation denoted by h_{t+1}. The expenditures for the education sector are financed by taxing wages of adult workers. Following Mankiw et al. (1992) by assuming that human capital and raw labor are perfect substitutes allows us to write aggregate human capital employment as $H_t = L_t h_t$.

2.2 Consumption side

Suppose that adults maximize their discounted lifetime utility determined by consumption in adulthood and after retirement in the vein of Diamond (1965)

$$\max_{c_t, s_t} u_t = \log c_t + \beta \log(R_{t+1}\ s_t), \tag{1}$$

where c_t denotes consumption, s_t represents savings carried over to retirement, $\beta = 1/(1+\rho)$ refers to the discount factor with ρ being the discount rate, and R_{t+1} denotes the gross interest rate paid on assets between generations t and $t+1$. Note that each time period corresponds to one generation and therefore lasts for 25 years. Assuming full depreciation of capital over the course of one generation, the gross interest rate corresponds to the capital rental rate and is given by $r_{t+1}+1$ with r_{t+1} being the net interest rate. The budget constraint of a young adult reads

$$(1-\tau)w_t h_t + l_t = c_t + s_t, \tag{2}$$

where τ denotes the income tax rate, w_t represents the wage per efficiency unit of labor and l_t are lump-sum redistributions of the monopolistic rents accruing in the intermediate goods sector after a patent has expired (see section 2.3.3 for details). Consequently, the left hand side of the budget constrained refers to total lifetime income of an individual which can be spent on consumption during adulthood or consumption after retirement. The results of the maximization problem are expressions for optimal consumption and savings

$$c_t = \frac{l_t + (1-\tau)h_t w_t}{1+\beta}, \tag{3}$$

$$s_t = \frac{\beta\left(l_t + (1-\tau)h_t w_t\right)}{1+\beta}, \tag{4}$$

exhibiting the standard properties for logarithmic utility, that is, they are increasing in wage income and lump-sum governmental transfers and decreasing in tax rates and the discount factor because the latter reduces savings and thereby lifetime interest income.

2.3 Production side

This subsection describes the production structure in the four sectors: final goods production, intermediate goods production, R&D and education. The treatment of the former two sectors is fairly standard (cf. Romer, 1990; Jones, 1995a; Strulik et al., 2011) and the description can be brief. However, we augment the standard framework to account for an income tax financed public education sector that employs labor to produce human capital of individuals and thereby increases the productivity of subsequent generations. Consequently, the education sector and the R&D sector compete for talents on the labor market.

2.3.1 Final goods sector

Final output Y_t being consumed by the adults and retirees in the economy and representing the gross domestic product (GDP) is produced according to the production function

$$Y_t = H_{t,Y}^{1-\alpha} \sum_{i=1}^{A_t} x_{t,i}^{\alpha}, \tag{5}$$

where $H_{t,Y}$ is human capital employed in the final goods sector, A_t is the technological frontier, that is, it represents the most modern blueprint that has been developed in the R&D sector, $x_{t,i}$ is the amount of the blueprint-specific machine i used in final goods production and α is the elasticity of final output with respect to machines. Due to perfect competition in the final goods market, production factors are paid their marginal products such that the wage rate per unit of human capital and prices of blueprints are determined as

$$\begin{aligned} w_{t,Y} &= (1-\alpha) H_{t,Y}^{-\alpha} \sum_{i=1}^{A_t} x_{t,i}^{\alpha} = (1-\alpha)\frac{Y_t}{H_{t,Y}}, & (6) \\ p_{t,i} &= \alpha H_{t,Y}^{1-\alpha} x_t^{\alpha-1}. & (7) \end{aligned}$$

Note that the derived prices for machines rely on the property that individual intermediate goods producing firms are deemed to be small in comparison to the whole sector. Consequently, the contribution of one such firm to the output of the whole sector can be neglected.[2]

2.3.2 Intermediate goods sector

We assume that a single intermediate goods producer is able to convert capital $k_{t,i}$ one for one into machines $x_{t,i}$ after it has purchased the corresponding blueprint from the R&D sector. Therefore its operating profits

[2] Sometimes an integral is used instead of the sum in equation (5) to capture this issue.

read

$$\pi_{t,i} = p_{t,i}k_{t,i} - R_t k_{t,i} \tag{8}$$

and profit maximization leads to the familiar outcome of Dixit and Stiglitz (1977) that firms charge prices for machines that are a markup $1/\alpha$ over marginal cost. Hence we have

$$p_{t,i} = \frac{R_t}{\alpha} \tag{9}$$

and we there is symmetry between firms such that the index i can be dropped. As another consequence of symmetry, we know that each firm employs $k_t = K_t/A_t$ units of capital, where K_t denotes the aggregate capital stock. Consequently, the aggregate production function reads

$$Y_t = (A_t H_{t,Y})^{1-\alpha} K_t^{\alpha}, \tag{10}$$

where technology is human capital augmenting.

2.3.3 R&D sector

The R&D sector employs scientists with a human capital level $H_{t,A}$ and productivity δ in order to develop new blueprints. The production function of a firm in the research sector can be written as

$$A_{t+1} - A_t = \delta A_t^{\phi} H_{t,A}, \tag{11}$$

where ϕ measures the extent of intertemporal knowledge spillovers. In case that $\phi = 1$ we would be in the Romer (1990) environment and sustaining an exponential growth rate of technology does not become ever more difficult as the technological frontier expands. We see from equation (11) that a constant amount of human capital in research would then suffice to have perpetual technological progress and therefore positive long-run economic growth. By contrast, if $\phi < 1$, we would be in the Jones (1995a) environment and a constant long-run growth rate of technology either requires a constant inflow of additional scientists into R&D, or a continuous increase in education of the scientists already employed, or both. Since we have positive population growth and human capital accumulation, no balanced growth path would exist in the Romer (1990) environment such that we assume $\phi < 1$ to hold from now on. Firms in the R&D sector maximize their profits

$$\pi_{t,A} = p_{t,A}\delta A^{\phi} H_{t,A} - w_{t,A} H_{t,A} \tag{12}$$

with $p_{t,A}$ being the price of a blueprint and $w_{t,A}$ being the wage rate of scientists. This leads to the optimality condition

$$w_{t,A} = p_{t,A}\delta A_t^{\phi}, \tag{13}$$

where wages of scientists increase in prices for blueprints.

It is assumed that patent protection for a newly discovered blueprint lasts for one generation. Afterwards the right to sell the blueprint is handed over to the government which redistributes the proceeds in a lump-sum manner. This assumption simplifies the exposition considerably and allows us tracing the transitional dynamics because in contrast to standard endogenous and semi-endogenous growth models, we do not require interest rates to remain constant over time (cf. Strulik et al., 2011, for a comparable mechanism). Therefore R&D firms can charge prices for blueprints that are equal to the operating profits of intermediate goods producers in time period t (when patent protection is valid) because there is always a potential entrant willing to pay that price. To put it differently, in case that blueprints were less (more) expensive, firms would have an incentive to enter (exit) the market and consequently the stable equilibrium involves zero overall profits. Therefore we can write prices for blueprints as

$$p_{t,A} = (\alpha - \alpha^2)\frac{Y_t}{A_t} \tag{14}$$

which follows from equations (7) and (9) and the fact that $x_i = k_i$ for all i.

2.3.4 Education sector

Finally, the education sector employs teachers financed by the proceeds of income taxes in order to produce human capital (cf. Gersbach et al., 2009, who use a comparable financing scheme for basic research in a hierarchical growth model.). We assume a balanced governmental budget such that we have

$$\tau w_t h_t L_t = w_t h_t L_{t,E}, \tag{15}$$

where the left hand side represents governmental revenues, that is, the proceeds of taxing the total wage bill $w_t h_t L_t$ in the economy, and the right hand side represents governmental expenditures, that is, the wages paid for teachers. This implies that the number of employed teachers is $L_{t,E} = \tau L_t$. Next, we assume that the education sector produces schooling intensity, denoted by e_t, according to

$$e_t = \xi \frac{L_{t,E}}{nL_t} = \xi \frac{\tau}{n}, \tag{16}$$

where ξ measures the productivity of teachers and τ/n denotes the teacher-pupil ratio. This implies that the intensity of schooling increases in the productivity of teachers and in public educational investments per child. Note that, ceteris paribus, faster population growth lowers the teacher-pupil ratio and thereby the schooling intensity. We assume that schooling intensity plays a similar role in individual human capital formation as an increase in the years of schooling in the corresponding literature: Building upon Mincer (1974) and following Hall and Jones (1999), Bils and Klenow (2000) and Caselli (2005), schooling intensity translates into individual human capital according to $h_{t+1} = \exp\left[\tilde{\psi}\left(\xi\tau/n\right)\right] h_t$, where $\tilde{\psi}(\cdot)$ measures the extent to which it does. As regards the particular specification of this function, Bloom and Canning (2005) use a linear relationship, which is based upon evidence by Psacharopoulos (1994).[3] We follow their approach such that

$$h_{t+1} = \exp\left[\psi\left(\xi\frac{\tau}{n}\right)\right] h_t \tag{17}$$

with $\psi = const$. Altogether, equation (17) implies that if the government does not invest in education at all, human capital of the successive generation will be the same as those of their parents. This can be justified by the notion that, without formal education, people are observing and learning from their parents and peers (cf. Strulik et al., 2011, p. 8). Furthermore, if people would not observe and learn from others at all, the model would lack positive economic growth in equilibrium, which would be at odds with stylized facts of development in modern economies (cf. Acemoglu, 2009; Galor, 2011).[4]

[3]For an alternative specification see Hall and Jones (1999), who assume a piecewise linear function that takes different values for primary, secondary and tertiary education.

[4]Of course it can be questioned whether a positive economic growth rate can be sustained indefinitely facing scarce resources, a limited carrying capacity of the environment and bounded space on earth. However, we do not insist that our model holds for $t \to \infty$

2.4 Market clearing and the balanced growth path of the economy

Labor market clearing implies that the total amount of available human capital is either employed in the final goods sector, in the education sector, or in the R&D sector, that is, we have $h_t L_t = h_t (L_{t,E} + L_{t,A} + L_{t,Y}) \Rightarrow H_t = H_{t,E} + H_{t,A} + H_{t,Y}$. Furthermore, we know that wages in all sectors have to equalize such that $w_{t,E} = w_{t,A} = w_{t,Y}$, otherwise one or more sectors would not be able to attract any workers and the economy ended up in a corner solution. Equalizing expressions (6) and (13), using equation (14) and noting that employment in the education sector is τL_t, yields demand for workers in the final goods sector and in the R&D sector as

$$H_{t,Y} = \frac{A_t^{1-\phi}}{\alpha\delta}, \tag{18}$$

$$H_{t,A} = (1-\tau)H_t - \frac{A_t^{1-\phi}}{\alpha\delta}. \tag{19}$$

Recalling that $H_t = h_t L_t$ and $H_{t,E} = h_t L_{t,E}$, we see that an increase in the population size or in individual human capital immediately leads to more employment of aggregate human capital in education and in science. The latter fosters technological progress such that A_{t+1} rises by more than it would have otherwise. This in turn increases human capital employment in the final goods sector in generation $t+1$. Altogether the development of new blueprints can be described by

$$A_{t+1} = \delta(1-\tau)A_t^{\phi} h_t L_t - \frac{1-\alpha}{\alpha} A_t, \tag{20}$$

where the basic trade-off that public educational investments imply is the following: while increasing taxes poaches labor from the R&D sector to the education sector, it also increases human capital accumulation and therefore the productivity of the next generation's scientists.

Full depreciation of capital and capital market clearing imply that the aggregate capital stock of an economy in generation $t+1$ is equal to aggregate savings. Furthermore, goods market clearing ensures that aggregate consumption together with aggregate savings is equal to total output such

but that it represents a reasonable approximation for a certain period of time.

that

$$K_{t+1} = s_t L_t = Y_t - c_t L_t. \tag{21}$$

These identities can then be used to eliminate the lump-sum redistributions of the government to the households. After doing so, the equation governing the accumulation of aggregate capital reads

$$K_{t+1} = \frac{\beta}{1+\beta}\left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} K_t^{\alpha}. \tag{22}$$

Putting all information together, the system fully describing the equilibrium dynamics of our model economy is given by

$$\begin{aligned}
A_{t+1} &= \delta(1-\tau)A_t^{\phi} h_t L_t - \frac{1-\alpha}{\alpha} A_t, && (23)\\
h_{t+1} &= \exp\left(\frac{\psi\xi\tau}{n}\right) h_t, && (24)\\
L_{t+1} &= n L_t, && (25)\\
K_{t+1} &= \frac{\beta}{1+\beta}\left(\frac{A_t^{2-\phi}}{\alpha\delta}\right)^{1-\alpha} K_t^{\alpha}. && (26)
\end{aligned}$$

Note that these equations hold during the transition to the balanced growth path *and* along the balanced growth path itself. Making use of the definition of a balanced growth path, that is, that the growth rate of a variable does not change over time, we can derive the rate of technological progress as

$$g_A = [(g_h+1)(g_L+1)]^{\frac{1}{1-\phi}} - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right) n\right]^{\frac{1}{1-\phi}} - 1, \tag{27}$$

where g_j denotes the growth rate of variable j. For the aggregate capital stock it follows either from equation (26) or from inspection of the aggregate production function that its long-run balanced accumulation rate is given by

$$\begin{aligned}
g_K &= (g_h+1)(g_L+1)(g_A+1) - 1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right) n\right]^{\frac{2-\phi}{1-\phi}} - 1\\
&= (g_A+1)^{2-\phi} - 1. && (28)
\end{aligned}$$

Denoting per capita GDP by y_t and putting everything together, the growth rates of aggregate GDP and per capita GDP are, respectively

$$g_Y = (g_h+1)(g_L+1)(g_A+1)-1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right)n\right]^{\frac{2-\phi}{1-\phi}} - 1, \quad (29)$$

$$g_y = (g_h+1)(g_A+1)-1 = \left[\exp\left(\frac{\psi\xi\tau}{n}\right)\right]^{\frac{2}{1-\phi}} n^{\frac{1}{1-\phi}} - 1. \quad (30)$$

Technological progress is driven by growth in aggregate human capital which is composed of individual human capital and the population size. It might seem that a decrease in both of these variables decreases the long-run growth rate of the economy. This, however, misses the point that human capital accumulation is inversely related to the population growth rate via the latter's negative influence on the teacher-pupil ratio. The question which of the two effects prevails will be discussed in Proposition 1.

Note that per capita GDP, the crucial measure for prosperity in growth theory, not only increases with the rate of technological progress but also with the rate of individual human capital accumulation. The whole process is then complemented by physical capital accumulation which ensures a constant capital-labor ratio and positive growth of per capita GDP even in the long run. Therefore the balanced growth path of the model is consistent with the stylized facts of economic development expressed by Kaldor (1957). Now we can state the first central analytical result of our paper.

Proposition 1. *The long-run growth rates of technology and per capita GDP decrease in response to faster population growth if the education sector of an economy is well-developed and the population growth rate is low. The converse holds true for an economy with fast population growth and a badly developed education sector.*

Proof. We take the derivatives of the growth rate of technology and per capita GDP with respect to population growth

$$\frac{\partial g_A}{\partial n} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right)n\right]^{\frac{1}{1-\phi}}(n-\xi\tau\psi)}{n^2(1-\phi)}, \quad (31)$$

$$\frac{\partial g_y}{\partial n} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right)n\right]^{\frac{2-\phi}{1-\phi}}(n-\xi\tau(2-\phi)\psi)}{n^3(1-\phi)}. \quad (32)$$

The first expression is negative if the education sector — as measured by the product of public investments in education represented by taxes (τ), productivity of teachers (ξ), and the Mincerian coefficient measuring the translation of the schooling intensity into human capital (ψ) — is in good shape, while the population growth rate (n), is low. Qualitatively the same result holds true for the growth rate of per capita GDP. □

The economic intuition behind these results is that growth of aggregate human capital is either due to growth of individual human capital or due to growth of the population size. An increase in population growth, which — by itself — positively impacts upon aggregate human capital accumulation, simultaneously decreases the teacher-pupil ratio. This in turn has a negative impact on the evolution of aggregate human capital. If the education sector is well developed and the population growth rate is low, the negative effect will dominate and population growth negatively impacts economic growth. This is most likely to be the case for developed countries, which would be consistent with the evidence found by Brander and Dowrick (1994), Kelley and Schmidt (1995), Ahituv (2001) and Bernanke and Gürkaynak (2001). If, on the other hand, the education sector is badly developed and population growth is high, the positive effect will dominate and population growth positively impacts economic growth.

Another interesting aspect is that the proof of Proposition 1 indicates that there exists a certain parameter range for which technological progress negatively depends on an increase in population growth, while the converse holds true for per capita output growth. The mathematical reason is that the negative part in the numerator is multiplied by $2-\phi > 1$ in the derivative of g_y with respect to n. The intuitive explanation is that individual human capital accumulation not only exerts its positive growth effect via the R&D sector but additionally through the channel suggested by Lucas (1988), that is, it increases productivity of workers in the final goods sector. Since faster accumulation of human capital is accompanied by faster physical capital accumulation, constant returns with respect to these two production factors on the aggregate level imply an additional positive impact of education on output growth. Now we turn to the second central analytical result of our paper.

Proposition 2. *The long-run growth rates of technology and per capita GDP unambiguously increase in public educational investments.*

Proof. We take the derivatives of the growth rate of technology and per capita GDP with respect to the tax rate

$$\frac{\partial g_A}{\partial \tau} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right)n\right]^{\frac{1}{1-\phi}}\xi\psi}{n(1-\phi)}, \tag{33}$$

$$\frac{\partial g_y}{\partial \tau} = \frac{\left[\exp\left(\frac{\psi\xi\tau}{n}\right)n\right]^{\frac{2-\phi}{1-\phi}}\xi(2-\phi)\psi}{n^2(1-\phi)}. \tag{34}$$

Since both of them are unambiguously positive, the proposition holds. □

This result is different compared to standard semi-endogenous growth models (cf. Jones, 1995a; Kortum, 1997; Segerström, 1998) because it suggests scope for economic policy to influence the long-run economic growth rate. This would rather be in line with the second wave of scale-free economic growth models advocated by Dinopoulos and Thompson (1998), Peretto (1998), Young (1998), Howitt (1999) and Dalgaard and Kreiner (2001). The policy measure to be taken is to increase investments in public education. In this regard our model is consistent with the literature suggesting a positive association between education and economic growth (cf. Barro, 1991; Sachs and Warner, 1995; Bils and Klenow, 2000; Krueger and Lindahl, 2001; Lutz et al., 2008). The reason for this effect to prevail is that — in the long-run and for a constant population growth rate n — there is only a positive effect of increasing education on aggregate human capital accumulation. Hence, in the long run, effective labor unambiguously grows faster in all sectors of the economy if the government raises educational investments. However, in the short- and medium run, that is, during the transition to the new balanced growth path, there could also be negative growth effects of increases in public educational investments because the education sector draws labor from the R&D sector. This represents the "near term costs" that Lutz et al. (2008) mention and which could be substantial. We turn to this issue in the next section, where we simulate an increase in educational expenditures and therefore keep track of the short- and medium-term costs as well as of the long-term benefits.

3 Simulating an increase in public educational expenditures

To address the question how the model economy is affected by an increase in public educational expenditures in the short- and medium run, we simulate the dynamic system displayed in equations (23) to (26) in the software package developed by Diks et al. (2008). The parameter values and justifications for using them are given in Table 2. We try to choose parameters to be consistent with data on the growth process of the United States obtained from World Bank (2012) or otherwise to be in line with the corresponding literature. Nevertheless, this is not an attempt to calibrate the model for a specific country which would be futile having time intervals of 25 years and a highly stylized framework. However, we aim to present a reasonably justified picture of the short- and medium-run response to an increase in public educational investments.

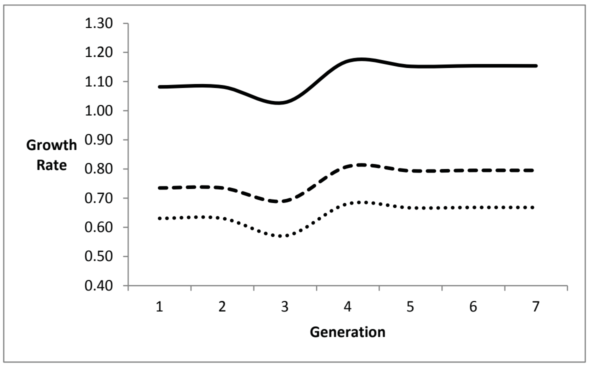
Table 2: Parameter values for simulation

Parameter	Value	Justification
β	0.3	Value implies a yearly discount rate of 5%
α	0.33	Value is common in the growth literature; see for example Jones (1995a)
δ	10	Parameter is free to choose; it changes the magnitude of the response to shocks during the transition period
ξ	10	The parameter values for ξ and ϕ imply
ϕ	0.5	g_y consistent with World Bank (2012) data for the United States
τ	0.0819	Value is implied by World Bank (2012) data for the United States
ψ	0.091	Value is commonly used/inferred; see for example Psacharopoulos (1994) and Bloom and Canning (2005)
n	1.2	Value implies population growth of 0.7% per year

The results of doing so are depicted in Figure 1. We assume that the economy initially moves along the balanced growth path. At generation 3 a 1 percentage point increase in public educational expenditures as a fraction of

GDP occurs. Afterwards the behavior of the economy is traced for another four generations, that is, for 100 years.

Figure 1: Simulation of an increase in public educational expenditures



Note: Time is displayed on the x-axis and growth (between two generations) is displayed on the y-axis. The solid line refers to capital, the dashed line to per capita output and the dotted line to technology. Initially, the economy moves along the balanced growth path. At the third generation, a 1 percentage point increase in public educational expenditures as a fraction of GDP occurs. Afterwards the economy is traced for another four generations, that is, for 100 years.

We see that the effect of an increase in public educational investments at impact is such that labor is drawn away from the R&D sector into the education sector which slows down technological progress (dotted line), per capita GDP growth (dashed line) and aggregate capital accumulation (solid line) for one generation. This reflects the "near term costs" of education that Lutz et al. (2008) mention in their article. In the subsequent generation, when the better educated workforce enters the labor market, the growth rate of technology, per capita GDP and the aggregate capital stock reach a peak. This is due to an upward level shift of aggregate human capital *and* to faster growth of individual human capital (cf. Trimborn et al., 2008, for the transitional effects of level shifts in standard semi-endogenous growth models). Afterwards the growth rates of technology, per capita GDP and aggregate capital converge to their new balanced growth path and they are higher than before the increase in educational investments. This is consistent with the claim expressed in Proposition 2.

4 Discussion

We set up an R&D-based economic growth model and extend it to allow for a *public* education sector. First, this allows us to generalize the R&D-based growth literature to take education, which is an empirically important determinant of economic development (cf. Barro, 1991; Sachs and Warner, 1995; Bils and Klenow, 2000; Krueger and Lindahl, 2001; Lutz et al., 2008), into account. We show that the long-run growth rate of the economy is not only affected by technological progress (being itself driven by population growth and human capital investments) but is further enhanced by sustained increases in the skills of the labor force together with faster physical capital accumulation. Consequently, the framework is able to bridge the gap between growth models relying solely on human capital accumulation like Lucas (1988) and the R&D-based growth literature. Second, we show that the long-run growth rates of technology and per capita output are sensitive to changes in governmental education policies. Therefore we challenge a property of early semi-endogenous growth models in the vein of Jones (1995a), Kortum (1997) and Segerström (1998) in favor of later scale-free growth models in the spirit of Dinopoulos and Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999). Third, our model framework suggests that increases in population growth might harm long-run economic growth perspectives in case that the education sector of an economy is well developed and population growth is slow. This primarily applies to industrialized countries in the second half of the twentieth century and therefore has the potential to explain the negative correlation between economic growth and population growth found in empirical studies for this time frame (cf. Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Bernanke and Gürkaynak, 2001).

From an applied perspective, our results suggest that educational investments are very important to foster long-run economic development. However, there might be substantial short- and medium-run costs associated with the implementation of growth promoting education reforms because resources from other sectors have to be transfered to the education sector. Moreover, the benefits of education do not materialize immediately because it takes time until the next generation enters the labor market. This essentially pins down to the trade-off between benefiting future generations at the expense of currently tax paying adults.

As already indicated, some aspects of the results in our paper have been shown within other frameworks. In particular, the notion that long-run economic growth is not solely driven by exogenously given population growth was the main reason for integrating horizontal and vertical innovations to remove the scale effect in otherwise endogenous growth models (cf. Dinopoulos and Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999). Moreover, *private* educational investments represent a main driving force behind long-run economic development for example in Dalgaard and Kreiner (2001) and Strulik et al. (2011). However, we are confident that our paper i) represents a consistent framework for analyzing these issues and their interrelations simultaneously, ii) sheds some light on the notion and importance of public education and especially the connection between schooling intensity, teacher-pupil ratios and population growth, and iii) allows for a fairly general dependence between population growth and economic prosperity being consistent with the empirical evidence.

We also acknowledge that our framework is highly stylized and some important issues cannot be treated within its realms. Possible extensions might therefore reveal other aspects of the connection between economic growth, education and demography. For example, the population growth rate and private educational investments could both be endogenized along the lines of Strulik et al. (2011) to analyze potential feedback effects between (public and privately financed) education, fertility and the teacher-pupil ratio. In particular, this could prove to be a useful framework for analyzing the extent to which public and private education complemented one another in the course of the industrial revolution (cf. Mokyr, 2005; Galor, 2011).

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