Higher education and the fall and rise of inequality

by

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Abstract

We investigate the effect of higher education on the evolution of inequality. In so doing we propose a novel overlapping generations model with three social classes: the rich, the middle class, and the poor. We show that there is an initial phase in which no social class invests in higher education of their children, such that the evolution of inequality is entirely driven by the level of bequests. Once a certain income threshold is surpassed, the rich start to invest in higher education of their children, which partially crowds out bequests and thereby reduces inequality in the short run. The better educated children of the rich, however, enjoy higher incomes and inequality starts to rise again. As time goes by, the middle class and eventually also the poor start to invest in higher education, but now the increase in inequality is driven by different levels of education. As the economy proceeds toward a balanced growth path, educational differences between social groups and thus inequality decline again. We argue that (1) the proposed mechanism has the potential to explain the U-shaped evolution of inequality in rich countries in the second half of the 20th Century and the first decade of the 21st Century and (2) the currently observed increase in inequality is rather a transitory phenomenon.

The main force pushing toward reduction in inequality has always been the diffusion of knowledge and the diffusion of education.
(Thomas Piketty)

1 Introduction

With the publication of the book "Capital in the 21st Century" by Thomas Piketty, interest in the analysis of economic inequality rose to previously unknown levels. Shortly after the English version of Piketty's book came out in March 2014, the prestigious journal "Science" published a special issue on this topic ("The science of inequality", May, 23rd 2014) that includes contributions from the most distinguished researchers in the field.[1] The three flagship publications of the OECD on inequality and its economic consequences (OECD, 2008, 2011, 2015) and the fact that Piketty's book became a best-seller, further emphasize the importance of distributional issues not only for scientists in the fields of economics, psychology, and sociology, but also in the general public debate and among policy makers.

As far as the empirical evidence for industrialized countries is concerned, Figure 1 displays the income shares (excluding capital gains)[2] of the richest 10% for the United States, Germany, and the United Kingdom from 1917 until 2013 using the data of "The World Top Incomes Database" compiled by Alvaredo et al. (2015).[3] We observe that inequality was high in the first half of the 20th Century with the richest 10% earning around 40% of total income in all three countries. Until World War II this share stayed relatively constant in the United Kingdom, it increased in the United States to a level of 45%, and it dropped in Germany until around 1930 after which it rose again to 38% until the eve of World War II. After 1945, there was a strong decrease in all three countries with the income share dropping to a level slightly above 30% and hovering there until around 1980. In the 1980s the income share of the richest 10% started to increase in the United States and in the United Kingdom (see also Piketty and Saez, 2003; Atkinson et al., 2011), while it remained relatively stable in Germany until the late 1990s (see also Dell, 2005). In the beginning of the 21st Century, however, it also started to rise in Germany. Altogether, the income share of the richest 10% is now close to or even above the level observed in the beginning of the 20th Century in all three countries.

The most salient features of this observed development are i) high initial inequality in all countries at the beginning of the 20th Century; ii) a substantial drop of inequality after

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[1] See Autor (2014), Deaton (2014), Haushofer and Fehr (2014), Piketty and Saez (2014), and Ravallion (2014) for detailed discussions of different aspects related to inequality.

[2] Considering also capital gains reduces the availability of the data. For the United States, where detailed data are available, this would, however, only lead to a more pronounced U-shaped pattern of inequality with respect to time.

[3] We are well aware of the problems with different measures of inequality (**?**). For the sake of data availability we use the top 10% income share to illustrate our motivation. Note, however, that top income shares also tend to be highly correlated with other measures of income inequality such as the Gini index.
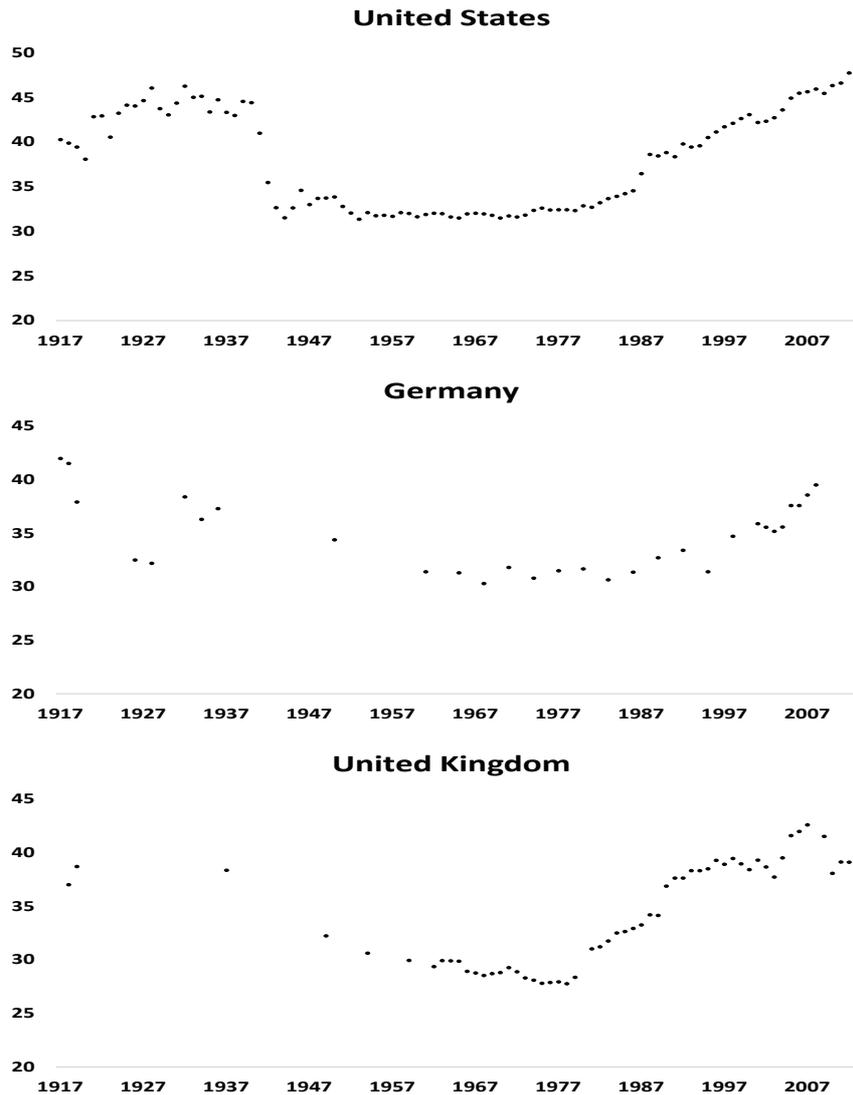
Figure 1: Share of total income of the richest 10% in the United States, Germany and the United Kingdom (excluding capital gains).

World War II;[4] iii) a relatively constant level of inequality throughout the 1950s, 1960s, and 1970s, i.e., for around one generation; and vi) strongly rising inequality thereafter.[5] Many explanations for this pattern have been proposed: the disruptions of World War II had a negative impact on wealth that, naturally, disproportionately impacted on the rich; substantial inheritance taxes and high marginal income tax rates for high incomes in

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[4]Parts i) and ii) are also referred to as the Kuznets curve relationship according to which inequality rises with increasing income in earlier stages of economic development, whereas it falls with income in later stages (Kuznets, 1955).

[5]The available data series for Japan start in 1947 such that i) and ii) cannot be assessed. The top 10% income share, however, rose from around 30% throughout 1947-1978 to 40.5% in 2010, which is in line with iii) and iv). France is an exception among the large industrialized countries in the sense that the pattern of the evolution of inequality followed i) and ii), but inequality did not rise substantially in the 1980s, 1990s, and 2000s (see also Piketty, 2003).

the 1950s, 1960s, and 1970s to cope with the debt burden left by World War II have also reduced the relative incomes of the rich. As far as the increase in inequality in the 1980s is concerned, changing tax policies (lowering marginal income tax rates and inheritance taxes) by the Reagan administration in the United States and the Thatcher government in the United Kingdom are another potential reason; declining population growth could have exacerbated the effect because bequests have to be divided amongst fewer heirs; skill-biased technological change disproportionately benefited the well-educated and led to an increase in the wage differential between low-paid and high-paid individuals; the decline of unionization decreased the bargaining power of low-paid workers and reinforced the increase in the wage differential; and globalization put further pressure on low incomes because low-skilled labor intensive production has often been relocated to low-income countries (see Acemoglu, 2002; Elsby et al., 2013; Piketty, 2014, for different arguments).

We propose a complementary mechanism, namely the increase in costly higher education after World War II. The most wealthy groups in different societies were the first to be able to invest massively in higher education of their children. At the same time this slows down intergenerational wealth accumulation among the rich because household's resources are re-allocated away from bequests (physical capital accumulation) toward higher education (human capital accumulation). The increase in the skills of the children of the rich raises their income once that they enter the labor market and also leads to a steeper age-income profile later on in their lives. The fact that a substantial amount of time elapses between education and labor market entry implies that there is a certain delay during which income inequality is reduced because of lower bequests. After a certain amount of time, however, inequality starts to rise again because the children of richer households are better educated and therefore earn higher (and faster growing) incomes. Altogether, we show that our mechanism is likely to have contributed to a decline in the top income share with a rebound after around one generation. Consequently, the proposed mechanism has the potential to explain (part of) the observed evolution of inequality in industrialized countries in the second half of the 20th Century and in the first decade of the 21st Century.

We conceptualize the outlined mechanism by assuming that there are three social classes, the rich, the middle class, and the poor. Initially, economic development and inequality are both driven by the accumulation of physical capital because the rich are able to save more due to subsistence consumption needs and lower incomes of the other two social classes. During this stage of economic development, which we call the neoclassical regime, investments in higher education are not yet widespread because higher education for the children is seen as a luxury good from the perspective of households. However, at some point in time an income threshold is surpassed, above which the rich start to invest in higher education of their children and correspondingly they reduce the savings rate and therefore the accumulation of wealth (physical capital accumulation). During this phase, which we call the high-skill regime, inequality first declines because high-skilled human capital accumulation of the children is costly for the parents, while the children do not yet supply their skills on the labor market. This phase of declining inequality lasts for

around one generation. Afterwards inequality rises again because the better educated children of the rich earn higher incomes, are thus able to bequeath higher levels of wealth to their offspring and are also able to invest more in their children's education than the less well educated children of the middle class and the poor. Subsequently, however, also the income levels of the middle class and potentially also those of the poor surpass the threshold levels above which investments in their children's higher education becomes a utility maximizing strategy. Since different social groups accumulate human capital at different rates, inequality starts to increase again. This mechanism gives rise to a candidate explanation of the observed U-shaped evolution of inequality, which we call the "Piketty curve", but in contrast to Piketty, we argue that the top income share does not approach 1 in the long-run. If all social classes manage to transit to the high skill regime, the growth rates of human capital converge between the rich, the middle class, and the poor in the long run, such that inequality declines again and the top income share moves to a level substantially below 1. Crucial for the emergence of declining inequality, though, is that all social classes switch to the high-skill regime. As we show, this is by no means guaranteed because the poor essentially face a risk of being disconnected from the growth process.

As far as the evolution of inequality an differential increase in higher educational attainment between the different income groups is concerned, the predicted pattern of our model is consistent with the data as reported by The Pell Institute (2015) for the United States. They show that, in 1970, 40% of the dependent members of families in the top income quartile had a Bachelor's degree by the age of 24. This number almost doubled to 77% in 2013. In the second highest income quartile, 15% of dependent family members had a Bachelor's degree in 1970, which more than doubled to 34% in 2013. In the lowest income quartile, however, only 6% of the dependent family members had a Bachelor's degree in 1970 and the number barely rose over time. In 2013 still only 9% of the dependent family members in the lowest income quartile are holding a Bachelor's degree.

The following articles are closely related conceptually and/or content-wise. Galor and Moav (2006) analyze the historical demise of the capitalist-worker class structure in an overlapping generations model with intergenerational transfers and public education. They show that the increasing importance of human capital in production after the Industrial Revolution led to a cooperation between capitalists and workers in the sense that capitalists started to support public education for the masses. The central reason for doing so is the complementarity between skills and physical capital in the sense that a better educated workforce raises the rate of return on physical capital. In a related paper, Galor et al. (2009) show how inequality in landownership adversely affected the emergence of institutions that promote basic education during the Industrial Revolution. The reason is that, due to a lower degree of complementarity between human capital and land (as compared to human capital and physical capital), landowners are interested in a reduction of the mobility or rural workers who might otherwise move to cities and work in factories. Consequently, they oppose mass education. In countries, where landownership is very

unequal, the theory of Galor et al. (2009) implies that the adverse effect of education on landowners is very strong such that they have a stronger incentive to oppose mass education. Altogether and consistent with the empirical evidence, countries with a higher inequality in landownership got surpassed by countries with a lower degree of inequality of landownership during the process of industrialization. These frameworks successfully explain patterns in the emergence of publicly funded basic education for the masses and the connection between education and inequality from a historical perspective. Galor and Zeira (1993) show that initial wealth disparities matter for the long-run distribution of incomes if capital markets are imperfect and there are indivisibilities in education investments. In so doing they add another explanation for the emergence of a Kuznets curve, i.e., increasing inequality first and then decreasing inequality in the course of economic development. In our paper we are, however, more concerned with the connection between higher education, which gained huge importance in the second half of the 20th Century, and the patterns of inequality after WW II as described by Piketty (2003) and Piketty (2014). We therefore focus on the next phase of increasing inequality throughout the period 1980-2010 and also show that, under certain circumstances, inequality might decline again in the future.[6]

The paper is organized as follows: Section 2 describes the model, Section 3 analyzes the dynamics, Section 4 illustrates the model solution by means of a numerical example, and Section 5 concludes and describes potential policy measures to reduce income inequality and its negative economic effects.

2 The model

Consider a small open economy populated by a continuum of overlapping generations. Time is discrete, indexed by t, and ranges from 0 to ∞.

A large number of firms produce aggregate output Y_t with physical capital K_t and human capital $H_{Y,t}$ using a constant returns to scale technology of the Cobb-Douglas type

$$Y_t = A K_t^\alpha H_{Y,t}^{1-\alpha}, \tag{1}$$

with $A > 0$ being total factor productivity (TFP) and $\alpha \in (0,1)$ denoting the elasticity of output with respect to physical capital. Perfect competition implies that equilibrium rates of reward are given by

$$w_t = (1-\alpha)Ak_t^\alpha, \qquad r_t + 1 = \alpha Ak_t^{\alpha-1}, \tag{2}$$

where $k_t = K_t/H_{Y,t}$ represents physical capital per unit of effective labor and there is

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[6]In an interesting article, Galor and Tsiddon (1997) focus on a different question and investigate the interactions between technological inventions and inequality. They show that periods with a lot of inventions go hand in hand with high inequality but also high intergenerational mobility, while periods of innovations are associated with lower inequality but also with lower intergenerational mobility.

full depreciation of physical capital over the course of one generation. Moreover, the interest rate is determined at the world capital market such that $r_t = \bar{r} = const.$ and $r + 1 \equiv R = \alpha A k^{\alpha-1}$, implying an equilibrium capital stock per unit of effective labor of

$$k = \left(\frac{\alpha A}{R}\right)^{\frac{1}{1-\alpha}} = const. \tag{3}$$

This in turn implies that the wage rate per unit of effective labor is constant as well and given by

$$w_t = w = (1-\alpha)A\left(\frac{\alpha A}{R}\right)^{\frac{\alpha}{1-\alpha}} = const. \tag{4}$$

Each member of generation t belongs to one out of three social classes that are indexed by $j = r, m, p$: the rich are referred to by r, the middle class by m, and the poor by p. The life-cycle of each individual consists of three distinct phases: childhood, adulthood, and retirement. Adults are endowed with one unit of time and $h_{j,t}$ units of human capital. They work, consume the amount $c_{j,t}$, give birth to $n > 0$ children, and potentially provide each child with $e_{j,t} \geq 0$ units of higher education. Moreover, adults save the amount $s_{j,t}$ to cover their own consumption needs in their last period of life, $c_{j,t+1}$, and to bequeath the amount $nb_{j,t+1}$ to their offspring.

We describe the lifetime utility of agent j, who was born in $t-1$, by using the utility function

$$u_{j,t} = \log(c_{j,t} - \bar{c}) + \gamma \log(h_{j,t+1}) + \beta\Big[\log(c_{j,t+1}) + \theta \log(b_{j,t+1})\Big], \tag{5}$$

where $\bar{c} > 0$ is the subsistence level of consumption, $\beta \in (0,1)$ represents the discount factor, $h_{j,t+1}$ refers to the level of human capital per child, $\gamma > 0$ denotes the utility weight that parents attach to the human capital level of their children, and $\theta \in (0,1)$ represents the utility weight of the bequests to each child. As usual, the log-linear specification assures analytical tractability. Note that our utility function is less restrictive than the usual specification because, due to the presence $\bar{c} > 0$, it allows the savings rate to depend on incomes. Population growth is treated as exogenous because we do not aim to analyze the quality-quantity trade-off, the demographic transition, and the take-off to long-run economic growth.[7] Instead, we assume that the economy already escaped the Malthusian stagnation in the past and is now industrialized. In the words of Hansen and Prescott (2002), the transition from Malthus to Solow has already occurred. What we are concerned with is the next transition from a physical capital based economy to an economy in which economic growth is primarily driven by human capital accumulation. To put it differently,

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[7]For an appropriate treatment of the historical take-off toward sustained economic growth see the Unified Growth Theory, in particular, Galor and Weil (2000), Jones (2001), Hansen and Prescott (2002), Galor and Moav (2002, 2006), Doepke (2004), Cervellati and Sunde (2005), Strulik and Weisdorf (2008), and Strulik et al. (2013). See Galor (2005, 2011) for detailed overviews of the literature and extensions to the baseline frameworks.

we are interested in the transition from Solow to Lucas (1988).

Higher education is financed by parents and provided by a schooling sector that employs lecturers, readers, and professors who are members of the middle class. For brevity we refer to them simply as lecturers from now on. Altogether, human capital evolves according to

$$
\begin{aligned}
h^h_{j,t+1} &= (\bar{e}_j + e_{j,t})\kappa h^h_{m,t} && \text{if } e_{j,t} > 0, && (6)\\
h^n_{j,t+1} &= \bar{e}_j, && \text{if } e_{j,t} = 0, && (7)
\end{aligned}
$$

where κ is the productivity of the education sector and $\bar{e}_j$ represents the baseline level of education that each child of each group j obtains, for example, by observing her parents and peers (see for example Strulik et al., 2013) or because it is provided costlessly by the community.[8] It is straightforward to assume that $\bar{e}_r \geq \bar{e}_m \geq \bar{e}_p$, i.e., that the rich do not acquire less baseline education than the middle class, which in turn does not acquire less baseline education than the poor.

If $e_{j,t} > 0$, the corresponding social class is in the high-skill regime as indicated by the superscript $l = h$ and otherwise it is in the neoclassical regime as indicated by the superscript $l = n$.[9] Consequently, the budget constraints for adults and retirees are given by

$$
\begin{aligned}
I^l_{j,t} &= \begin{cases} c^h_{j,t} + s^h_{j,t} + wh^h_{m,t}ne^h_{j,t}, & \text{if } e_{j,t} > 0,\\ c^n_{j,t} + s^n_{j,t}, & \text{if } e_{j,t} = 0, \end{cases} && (8)\\
s^l_{j,t} &= \frac{c^l_{j,t+1} + nb^l_{j,t+1}}{R}, && (9)
\end{aligned}
$$

with income being denoted by $I^l_{j,t} = wh^l_{j,t}(1 - zn) + Rb^l_{j,t}$, where $z \in (0,1)$ is the time share necessary to raise one child to adulthood.[10] Note that the term $Rb^l_{j,t}$ refers to the bequests (plus interest payments) that a member of the cohort born in $t-1$ gets from her parents.

The following lemma summarizes households' optimal decisions.

Lemma 1.

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[8] Note that our formulation is a special case of de la Croix and Doepke (2003, 2004) and Glomm and Ravikumar (1992): $h_{j,t+1} = (\bar{e}_j + e_{j,t})^\eta h^\nu_{j,t}\bar{h}^{1-\nu}_{j,t}$, where $\bar{h}_{j,t}$ denotes average human capital, ν the intergenerational transmission of human capital, and η the impact of education on human capital. We set $\eta = 1$ and $\nu = 0$ for notational convenience without affecting the generality of our results. Moreover, we allow the productivity of the education sector, κ, to differ from 1. Regarding the interpretation of $\bar{e}_j$ as public schooling note that a tax-financed level of $\bar{e}_j$ would just depend on the structural parameters of our model (see de la Croix and Doepke, 2004, for more details).

[9] To reduce the complexity in the notation, we omit the superscript whenever this is possible.

[10] Recall that lecturers are recruited from the middle class, such that the costs for higher education depend on $h^h_{m,t}$.

(i) If $l = n$, agents maximize (5) subject to (8) and (9), such that

$$c_{j,t}^n = \frac{I_{j,t}^n + [(1+\theta)\beta]\bar{c}}{1 + (1+\theta)\beta}, \tag{10}$$

$$c_{j,t+1}^n = \frac{\beta}{1 + (1+\theta)\beta}[I_{j,t}^n - \bar{c}]R, \tag{11}$$

$$b_{j,t+1}^n = \frac{\theta\beta}{n[1 + (1+\theta)\beta]}[I_{j,t}^n - \bar{c}]R, \tag{12}$$

with $e_{j,t} = 0$ implying that $h_{j,t+1}^n$ is constant according to (7).

(ii) If $l = h$, agents maximize (5) subject to (8), (9), and (6), such that

$$c_{j,t}^h = \frac{I_{j,t}^h + [\gamma + (1+\theta)\beta]\bar{c} + wh_{m,t}^h n\bar{e}_j}{1 + \gamma + (1+\theta)\beta}, \tag{13}$$

$$c_{j,t+1}^h = \frac{\beta}{1 + \gamma + (1+\theta)\beta}[I_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j]R, \tag{14}$$

$$b_{j,t+1}^h = \frac{\theta\beta}{n[1 + \gamma + (1+\theta)\beta]}[I_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j]R, \tag{15}$$

$$e_{j,t}^h = \frac{\gamma}{wh_{m,t}^h n[1 + \gamma + (1+\theta)\beta]}(I_{j,t}^h - \bar{c}) - \frac{[1 + (1+\theta)\beta]\bar{e}_j}{1 + \gamma + (1+\theta)\beta}, \tag{16}$$

with $h_{j,t+1}^h$ evolving according to (6).

We observe from Lemma 1 that, *ceteris paribus*, consumption and bequests increase with income ($I_{j,t}$), bequests decrease with population growth (n), and both second period consumption and bequests increase with the discount factor (β) and with the interest rate (R), whereas they decrease with the subsistence consumption level ($\bar{c}$). The existence of $\bar{c} > 0$ implies a hierarchy of needs for households: expenditure shares for first period consumption are declining with income, while expenditure shares on second period consumption and on bequests are increasing with income. Hence, richer households save more and bequeath more wealth to their children, which is a well-known fact and which is the driver of inequality in the neoclassical regime.

In the high-skill regime, educational investments are positive and decrease with the preference for bequests (θ), population growth (n), the discount factor (β), the subsistence consumption level ($\bar{c}$), and the baseline education level ($\bar{e}$), whereas they increase with the preference for education (γ). Moreover, in light of (16), we observe that the regime switch from the neoclassical to the high-skill regime occurs if and only if the level of income, $I_{j,t}^l$, is sufficiently high. This is expressed formally in the following proposition.

Proposition 1.
A member of social class $j = r, m, p$ invests in eduction, i.e., $e_{j,t} > 0$, if her income exceeds the critical threshold $I_{j,t}'$ defined as

$$I_{j,t}' = \frac{wh_{m,t}^h n\bar{e}_j}{\gamma}[1 + (1+\theta)\beta] + \bar{c}. \tag{17}$$

Hence, $e_{j,t} = 0$ if $I_{j,t}^n \leq I_{j,t}'$.

Regarding the critical level of income that induces the regime switch to the high-skill regime several remarks are in order. (1) The threshold level of income necessary to induce the regime switch depends positively on the level of subsistence consumption, $\bar{c}$, positively on the sum of the weights of first and second period consumption, $1 + \beta$, as well as on the weight of bequests, $\theta\beta$. A higher preference for education, γ, reduces in turn the critical income level. Moreover, $I_{j,t}'$ is increasing in the level of baseline education, $\bar{e}_j$, and education cost, $wh_{m,t}^h n$. (2) In light of Proposition 1, the high-skill regime applies to households of class j if their income level is sufficiently high such that investments in education above the basic level (that can be acquired costlessly) deliver a higher amount of additional utility as using the same amount of income for consumption and bequests. In this case we have $I_{j,t} > I_{j,t}'$, such that $e_{j,t} > 0$. The optimal solutions are then described by item (ii) of Lemma 1. If the income level of households in social class j falls short of a critical threshold $I_{j,t}'$, these households find it optimal not to invest in education of their children because the children acquire a certain amount of human capital costlessly and the additional investments in human capital would deliver less additional utility than if the same amount of income was instead spent on consumption or was bequeathed. These households find themselves in the neoclassical regime, i.e., $l = n$ and $e_{j,t} = 0$, such that their optimal decisions are described by item (i) of Lemma 1.[11] (3) As long as the middle class does not switch to the high-skill regime, $I_{j,t}'$ is constant. If the middle class starts to invest in higher education of their children, $I_{j,t}'$ is growing with the growth rate of $h_{t,m}$. If furthermore the middle class switches before the poor and after the rich, the described mechanism becomes crucial for the poor because of the risk that $I_{p,t}'$ is growing faster than their incomes such that a regime switch of the poor would require policy interventions in terms of publicly financed universities or higher education subsidies such as stipends. (4) In case of $\bar{e}_r = \bar{e}_m = \bar{e}_p$, it follows that all social classes exhibit the same threshold income but social classes switch at different dates depending on the distribution of wealth. Thus, even without any differences in the level of basic education among the social classes, there is a risk that the poor will be disconnected from the growth process in the sense that their incomes are lagging behind their critical income level $I_{p,t}'$. If, in turn, $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $I_{r,t}' > I_{m,t}' > I_{p,t}'$. This effect is, however, compensated because the rich attain a higher level of wealth and a higher accumulation rate of wealth, such that they reach their threshold level before the middle class, which in turn reaches its threshold level before the poor.

Taking into account the time required for child-care (zn), each social class supplies

$$L_{j,t} = (1 - zn)N_{j,t} \tag{18}$$

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[11]See also Solow (1956), Ramsey (1928), Cass (1965), Koopmans (1965), and Diamond (1965) for the analysis of capital accumulation in a neoclassical setting without human capital accumulation. The last reference is closest to our framework because of its discrete time overlapping generations formulation.

units of of raw labor, where $N_{j,t}$ is the number of individuals who belong to social class j. Thus, the aggregate supply of human capital can be obtained as the sum of human capital within each social class, which is in turn given by the product of class-specific labor supply ($L_{j,t}$) and average class-specific human capital ($h_{j,t}$):

$$H_t = h_{r,t}L_{r,t} + h_{m,t}L_{m,t} + h_{p,t}L_{p,t}. \tag{19}$$

Consequently, aggregate supply of human capital allocated to production is obtained as

$$H_{Y,t} = \begin{cases} H_t - (e_{r,t}N_{r,t} + e_{m,t}N_{m,t} + e_{p,t}N_{p,t})h_{m,t}L_{m,t}, & \text{if } l = h, \\ H_t, & \text{if } l = n. \end{cases} \tag{20}$$

This takes into account that human capital used for production is equal to total available human capital net of the human capital that is employed in the higher education sector, which is recruited from the middle class.

3 Dynamics

3.1 Neoclassical regime

In light of Proposition 1, the neoclassical regime is characterized by $I_{j,t} \le I'_{j,t}$, such that $e_{j,t} = 0$. Consequently, the level of human capital is constant and equal to the level of baseline education, i.e., $h^n_{j,t+1} = \bar{e}_j$. Thus, income of a member of social class j is given by

$$I^n_{j,t} = w\bar{e}_j(1 - zn) + Rb^n_{j,t}. \tag{21}$$

In the following, we denote stationary variables with an asterisk in the subscript. Given that labor incomes are stationary in the neoclassical regime, the accumulation of wealth via bequests is the only source of income growth. Income, $I^n_{j,*} = w\bar{e}_j(1-zn)+Rb^n_{j,*}$, is thus constant when bequests approach their steady-state level, $b^n_{j,*}$. This implies that a regime switch from the neoclassical regime to the high-skill regime can only occur if the threshold levels of income that are compatible with the regime switch are below their steady-state levels, i.e., a necessary condition for the regime switch is $I^n_{j,*} > I'_{j,t}$.[12] The following proposition specifies this aspect in more detail. A graphical illustration is presented in Figure 2.

Proposition 2.

(i) According to (12) and (21), the accumulation of wealth within social class j is gov-

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[12]Note that this would even be the case in the presence of technological progress fueled by other sources than human capital because disposable incomes and the threshold level $I'_{j,t}$ are growing at the same rate.

erned by

$$b^n_{j,t+1} = \frac{\theta\beta}{n[1 + (1+\theta)\beta]}[w\bar{e}_j(1 - zn) + Rb^n_{j,t} - \bar{c}]R \qquad (22)$$

with a unique and stable steady state at

$$b^n_{j,*} = \frac{\theta\beta R}{n[1 + (1+\theta)\beta] - \theta\beta R^2}[(1 - zn)w\bar{e}_j - \bar{c}], \qquad (23)$$

given that $\theta\beta R^2 < n[1 + (1+\theta)\beta]$.

(ii) *The regime switch requires* $I_{j,*} = (1 - zn)w\bar{e}_j + Rb_{j,*} > I'_{j,t}$, *which implies in light of (21) and (23) that*

$$b^n_{j,*} > \frac{wh_{m,t}n\bar{e}_j}{\gamma R}[1 + (1+\theta)\beta] + \frac{\bar{c}}{R} - \frac{(1 - zn)w\bar{e}_j}{R} \qquad (24)$$

and

$$\bar{e}_j > \hat{e}^h \equiv \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - wh^h_{m,t}[n(1 + (1+\theta)\beta) - \theta\beta R^2]\}} \qquad (25)$$

if $l = h$, *and*

$$\bar{e}_j > \hat{e}^n \equiv \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - w\bar{e}_m[n(1 + (1+\theta)\beta) - \theta\beta R^2]\}} \qquad (26)$$

if $l = n$.

With regard to item (i) in Proposition 2, labor income must exceed the level of subsistence consumption, i.e., $(1 - zn)w\bar{e}_j > \bar{c}$, otherwise the steady state is economically meaningless. Moreover, global stability of $b^n_{j,*}$ requires that $\theta\beta R^2 < n[1 + (1+\theta)\beta]$.[13] In addition, note that the location of the b^n_j-locus as defined by (22) depends positively on $\bar{e}_j$. If $\bar{e}_r > \bar{e}_m > \bar{e}_p$, it follows that $b^n_{r,*} > b^n_{m,*} > b^n_{p,*}$.

A transition into the high-skill regime [item (ii)] requires that the long-run value of bequests in social class j as given by $b^n_{j,*}$ is larger than the threshold level of wealth $b'_{j,t}$ that is associated with the threshold level of income $I'_{j,t}$. This is only possible if $\bar{e}_j > \hat{e}$. From item (i), we know that a feasible and globally stable steady state, $b^n_{j,*} > 0$, requires that $n[1 + (1+\theta)\beta] - \theta\beta R^2 > 0$. As $\gamma\bar{c} > 0$ and $\bar{e}_j > 0$, (25) and (26) hold only if $\gamma(1 - zn)w > \{n[1 + (1+\theta)\beta] - \theta\beta R^2\}$. If the middle class is in the neoclassical regime, the poor need a minimum $\bar{e}_p > \hat{e}^n$, otherwise their steady state income falls short of their threshold income. In case that the middle class already experienced a regime switch, costs of higher education are increasing such that the threshold level of income necessary to conduct the regime switch is also increasing. Unless the baseline level of education of the poor fulfills (25) the regime switch is still possible.

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[13]For $(1 - zn)w\bar{e}_j < \bar{c}$, a steady state exists only if $\theta\beta R^2 > n[1 + (1+\theta)\beta]$, which implies, in turn, that $b^n_{j,*}$ is globally unstable.
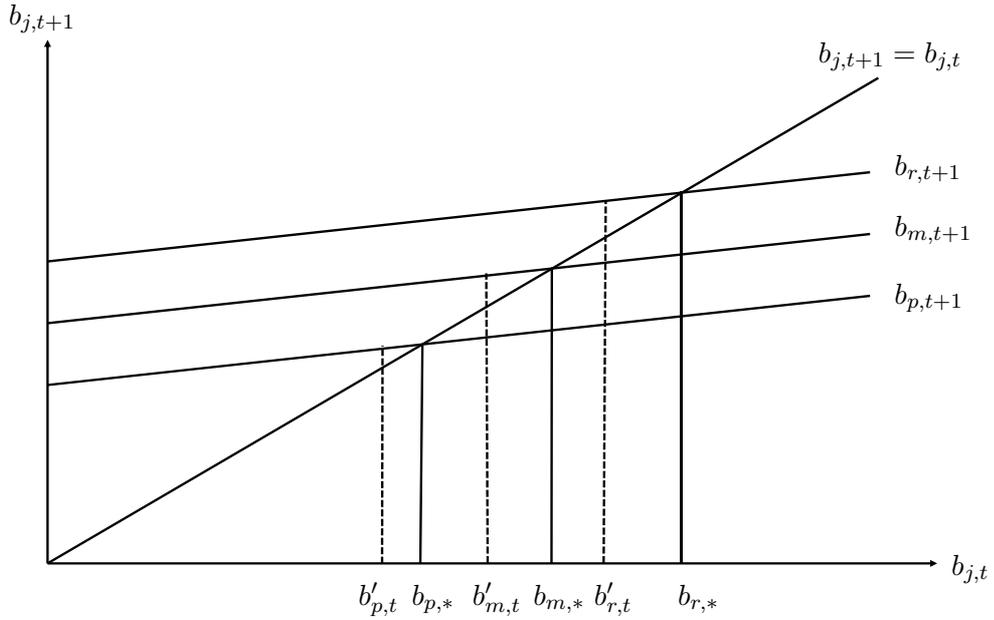
Figure 2: Neoclassical regime, with $\bar{e}_r > \bar{e}_m > \bar{e}_p$.

If $\bar{e}_j \leq \hat{e}^l$, accumulated assets of social class j sustained by the neoclassical regime fall short of $I'_{j,t}$ such that a switch to the high-skill regime cannot occur for social class j. If $\hat{e}^n > \bar{e}_r$, no social class would ever invest in higher education of their children. In this case the economy is trapped in the neoclassical regime as described by the Solow (1956) model and growth would cease at a certain point. This could be a relevant description of the "middle income trap", i.e., that countries, which successfully escaped the phase of stagnation at the subsistence level, cannot manage to switch to an innovation-based high-skill economy (cf. Eichengreen et al., 2012, 2013, who observe, among other things, that middle income traps are less likely to be an obstacle for countries with a well educated population). By contrast, a switch of all social classes to the growth regime is guaranteed for $\bar{e}_p > \hat{e}^l$.

3.2 High-skill regime

In the high-skill regime, income of social class j exceeds $I'_{j,t}$, such that these parents invest in higher education of their children. Their optimal decisions are represented by item (ii) of Lemma 1 and the evolution of human capital in social class j is then governed by (6). We summarize the dynamic behavior in the high-skill regime in the subsequent proposition.

Proposition 3.

(i) In the high-skill regime, the evolution of human capital and bequests is governed by

the following system of difference equations

$$h_{j,t+1}^h = (\bar{e}_j + e_{j,t})\kappa h_{m,t}^h, \tag{27}$$

$$b_{j,t+1}^h = \frac{\theta\beta}{n[1+\gamma+(1+\theta)\beta]}[wh_{j,t}^h(1-zn) + Rb_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j]R. \tag{28}$$

(ii) *The ratio between bequests and human capital is constant and the same for all social classes that switched to the high-skill regime, i.e.,*

$$\frac{b_{j,t+1}^h}{h_{j,t+1}} = \frac{w\theta\beta R}{\gamma\kappa} = const. \tag{29}$$

(iii) *Let $x_{j,t}^h$ denote the ratio of human capital between social class j and the middle class, i.e., $x_{j,t}^h = h_{j,t}^h/\bar{h}_{m,t}^h$, such that*

$$x_{j,t+1}^h = \frac{e_{j,t}+\bar{e}_j}{e_{m,t}+\bar{e}_m} = \frac{\left[(1-zn)+\frac{\theta\beta R^2}{\gamma}\right]x_{j,t}^h - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_j n}{(1-zn)+\frac{\theta\beta R^2}{\gamma} - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_m n}. \tag{30}$$

The stationary solution is given by

$$x_{j,*}^h = \lim_{t\to\infty} x_{j,t+1} = \bar{e}_j/\bar{e}_m, \tag{31}$$

with $x_{j,*}^h \gtreqqless 1$, if $\bar{e}_j \gtreqqless \bar{e}_m$.

(iv) *The gross growth rate of human capital is given by*

$$\frac{h_{j,t+1}^h}{h_{j,t}^h} = (\bar{e}_j + e_{j,t})\frac{h_{m,t}^h}{h_{j,t}^h} = \frac{\gamma\kappa\left[(1-zn)+\frac{Rb_{j,t}^h}{h_{j,t}^h} - \frac{\bar{c}}{wh_{j,t}^h} + \frac{n\bar{e}_j}{x_{j,t}^h}\right]}{n[1+\gamma+(1+\theta)\beta]} \tag{32}$$

$$= \frac{\gamma\kappa\left[(1-zn)+\frac{\theta\beta R^2}{\gamma} - \frac{\bar{c}}{wh_{j,t}^h} + \frac{n\bar{e}_j}{x_{j,t}^h}\right]}{n[1+\gamma+(1+\theta)\beta]}. \tag{33}$$

In the long-run it converges to

$$\lim_{t\to\infty}\frac{h_{j,t+1}^h}{h_{j,t}^h} = \frac{\gamma\kappa(1-zn+\bar{e}_m n) + \theta\beta R^2}{n[1+\gamma+\beta(1+\theta)]}. \tag{34}$$

The regime switch to the high-skill regime induces growing wage incomes, $wh_{j,t}^h$, and increasing levels of bequests [item (i) of Proposition 3], such that total incomes, $I_{j,t}^h$, grow as well. After the regime switch, the ratio between bequests and human capital is constant and the same for all social classes [item (ii)], such that incomes grow at the same rate. The ratio of bequests to human capital, $b_{j,t+1}^h/h_{j,t+1}$, declines with the weight of children's education in the parental utility function, γ, and the productivity of the education sector,

κ. On the other hand, it increases with the arguments responsible for an increase in bequests, i.e., wages, w, which also increase education costs, the weight of bequests in the parental utility function, $\theta\beta$, and the interest factor, R.

Inequality is determined by the initial distribution of wealth and by different levels of baseline education, $\bar{e}_j$. In item (iii), we capture the evolution of inequality in terms of education by the dynamics of the ratio between human capital of social class j and the level of human capital of the middle class, i.e., $x_{j,t} = h_{j,t}^h / h_{m,t}^h$. As regards the evolution of relative human capital, the cases of different or equal levels of baseline education, $\bar{e}_j$, should be distinguished. (1) If social classes do not differ with respect to their baseline levels of education such that $\bar{e}_r = \bar{e}_m = \bar{e}_p$, it follows in light of (30) that the influence of subsistence consumption on the evolution of $x_{j,t}$ is approaching zero in the long-run $(\lim_{t\to\infty} \bar{c}/wh_{j,t}^h = 0)$. Thus $x_{j,t}$ converges to 1 as t approaches infinity, if there are no differences in the baseline levels of education. Item (ii) implies then that all social classes bequeath the same amount of assets to their children, such that incomes are equal between social classes in the long run. Thus, there is only scope for long-run inequality if not all social classes switch to the high-skill regime. Transitory inequality is determined by the initial distribution of wealth for a given constellation of $I'_{j,t} < I_{j,*}^n$ determining the timing of the regime switch in social class j. Note, in this context, that the transition phase lasts for a very long time period (several generations) such that inequality could be observed for centuries if such a model represents the underlying data generating process. (2) If in turn $\bar{e}_j \gtrless \bar{e}_m$, it follows that x_j approaches $\frac{\bar{e}_j}{\bar{e}_m} \gtrless 1$ if $\bar{e}_j \gtrless \bar{e}_m$. Thus differences in the level of baseline education translate into differences in relative human capital endowments, different levels of bequests, and different levels of income. A constant $b/h-$ ratio implies then that households with lower human capital endowments exhibit also lower bequests, even in the long run.

The growth rate of human capital [item (iv)] depends positively on the $b/h-$ratio, positively on time devoted to work, and positively on $\gamma/\{n[1+\gamma+(1+\theta)\beta]\}$, which drives the expenditure share of higher education. Moreover, the growth rate of human capital is positively affected by the baseline level of education, $\bar{e}_j$, and the productivity of the education sector, κ. During the transition, the growth rate of human capital is adversely affected by subsistence needs, $\bar{c}$, and by $x_{j,t}^h$. The latter reflects a neoclassical convergence mechanism. In the long run, due to increasing wage incomes, the impact of subsistence needs on the evolution of human capital approaches zero. Altogether, $x_{j,t}$ converges to $\bar{e}_j/\bar{e}_m$, such that the growth factor of human capital converges to expression (34).

Crucial for our theory is the emergence of declining and then increasing inequality after the regime switch from the neoclassical to the high-skill regime. The main argument is summarized in the following proposition. The possible decline in inequality in a later phase of the high-skill regime will be discussed further below.

Proposition 4. *A social class that reaches the critical income level $I'_{j,t}$ reduces the accumulation of wealth.*
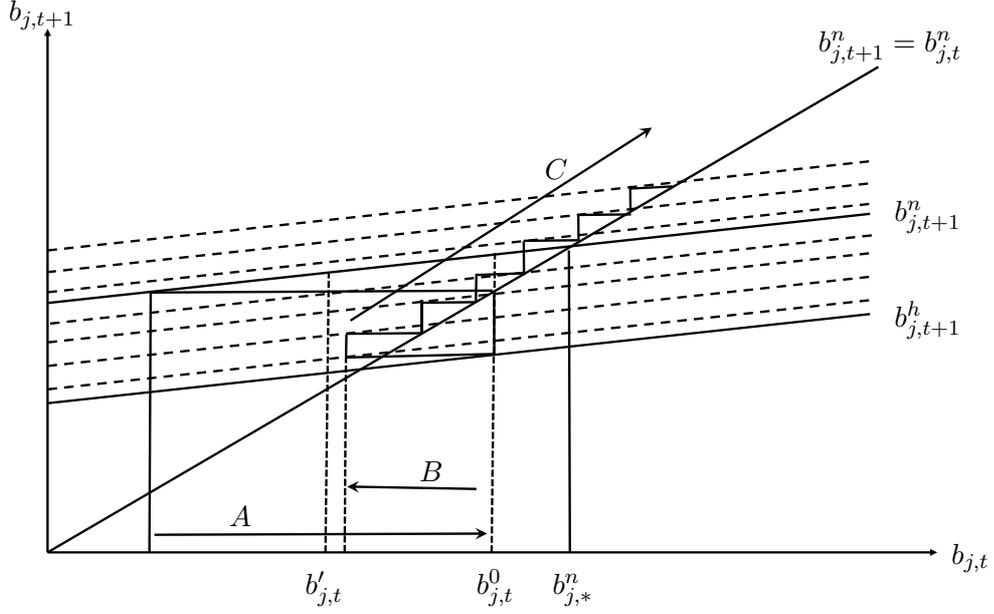
Figure 3: Regime switch to the high-skill regime

The intuition behind Proposition 4 is as follows: At the moment of the regime switch, a member of social class j is equipped with a wealth level of at least $b'_{j,t}$, which just assures the necessary level of income that generates the regime switch ($I'_{j,t}$, as defined by Proposition 1). In the neoclassical regime, according to Lemma 1, an income level of $I^0_{j,t}$ induces a level of bequests of

$$b^n_{j,t+1} = \frac{\theta\beta R}{n[1+(1+\theta)\beta]}[I^0_{j,t} - \bar{c}]. \tag{35}$$

In the high-skill regime the level of bequests is given by

$$b^h_{j,t+1} = \frac{\theta\beta R}{n[1+\gamma+(1+\theta)\beta]}[I^0_{j,t} - \bar{c} + w_t\bar{h}^h_t n\bar{e}_j], \tag{36}$$

From the last two expressions, we obtain $b^n_{j,t+1} > b^h_{j,t+1}$ if

$$I^0_{j,t} > [1+(1+\theta)\beta]\frac{wh^h_{m,t}n\bar{e}_j}{\gamma} + \bar{c}, \tag{37}$$

which, in light of Proposition 1, implies that $I^0_{j,t} > I'_{j,t}$. Thus, a household switching to the high-skill regime starts to invest in higher education of the children but at the expense of bequests per child. To put it differently, households shift resources from the accumulation of wealth in the form of physical capital to the accumulation of human capital. The rich are the first social class that experiences the regime switch and reduces bequests, followed by the middle class and possibly also the poor. Given that the expenditure share of

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bequests is, due to the existence of subsistence needs, increasing with income, the decline in bequeathed assets in the highest income percentiles may be outperformed by increasing levels of bequests in the other social classes, such that inequality may fall. As labor incomes of the children of the rich are increasing because of human capital accumulation (which stimulates again increasing levels of bequests and education), inequality may start to rise again. Altogether this follows the pattern of the "Piketty-curve" as described above. A graphical illustration of our arguments is presented in Figure 3. The evolution of bequests follows the arrows A, B, and C. In period t, members of social class j exhibit a level of assets indicated by $b_{j,t}^0$, which exceeds $b_{j,t}'$ associated with the threshold income $I_{j,t}'$. Thus, social class j switches to the high-skill regime and leaves the $b_{j,t+1}^n$-locus. From Proposition 4 it follows that the $b_{j,t+1}^h$-locus compatible with $b_{j,t}^0$ must be located below the $b_{j,t+1}^n$-locus, such that the level of bequests shrinks (B). Since the evolution of social class j is now described by Proposition 3, item (i), the $b_{j,t+1}^h$-locus moves upwards such that bequests increase again (C). Obviously, the speed of the upward shift depends (ceteris paribus) essentially on the quality of the education sector reflected by the productivity parameter κ. Thus, κ affects also the evolution of inequality.

Regarding the evolution of inequality after the regime switch it is important to note that the discussed fall and rise in inequality is just a transitory phenomenon. Relative human capital stocks will converge to their initial values and inequality will decline toward its long-run value. Nevertheless, and this will be clarified further below, the emergence of declining inequality and its potential amount depends crucially on whether or not all social groups experience a regime switch to the high-skill regime. In light of Lemma 1 expenditures for higher education are zero in the neoclassical regime, such that we obtain from Proposition 3, item (iii), initial relative human capital endowments in the neoclassical regime as

$$x_{r,t}^n = \frac{\bar{e}_r}{\bar{e}_m} > 1, \qquad x_{m,t}^n = 1, \qquad x_{p,t}^n = \frac{\bar{e}_p}{\bar{e}_m} < 1, \tag{38}$$

which are equal to the corresponding long-run values in the high-skill regime, $x_{j,*}^h$, [see (31)]. Due to a higher level of wealth and a higher convergence speed, the rich reach their critical threshold income first, followed by the middle class and possibly also by the poor. Higher education in the rich population group can be expressed as

$$e_{r,t} = \frac{\gamma}{n[1+\gamma(1+\theta)\beta]}\left[(1-zn)+\frac{R^2\theta\beta}{\gamma}\right]x_{r,t}^h - \frac{\bar{c}}{wh_{m,t}} - \frac{[1+(1+\theta)\beta]\bar{e}_r}{1+\gamma(1+\theta)\beta}. \tag{39}$$

With $h_{m,t}$ being constant, $x_{r,t}^h$ increases after the regime switch of the rich above $x_{r,t}^n$ and induces thus increasing expenditures on eduction, which amplifies the income gap to the other social classes later on. After the middle class experiences a regime switch, the resulting increase in $h_{m,t}$ affects relative endowments of the other social groups, while $x_{m,t}$ remains at 1. If the poor are still not investing in skills, their relative human capital stock shrinks and falls short of $x_{p,t}^n$. Moreover, their threshold income ($I_{p,t}'$) is now increasing

with the growth rate of human capital in the middle class. If, under these circumstances, $I'_{p,t}$ is increasing above $I^n_{p,*}$, the poor will never switch (or they may just temporarily switch) to the high-skill regime given that the growth rate of their incomes is below the growth rate of $I'_{p,t}$.

Initially $e_{r,t}$ increases because of the increase in $x^h_{r,t}$ indicating that education is comparatively cheap for the rich. When the middle class starts to invest in education, there is a dampening effect on $x_{r,t}$ because of the increase in education costs due to the increase in $h_{m,t}$, but a second reinforcing effect on education setting in through the diminishing role of subsistence needs, $\bar{c}/(wh_{m,t})$. The latter, however, is only a transitory effect that becomes smaller and smaller as the middle class accumulates human capital. Thus, the rich reduce the growth rate of expenditures on eduction below the level of the middle class such that $x^h_{r,t}$ converges from above to its long run value $x^h_{r,*} = x^n_{r,t}$. A symmetric argument holds for the poor. The regime switch of the middle class adversely affects the poor's relative human capital stock, $x_{p,t}$. Thus, their expenditures fall short of the level achieved in the middle class and $x^h_{p,t}$ shrinks below $x^n_{p,t}$. On the other hand, the declining importance of subsistence needs dampens the decline over time which implies also that the growth in expenditures on education in the middle class ceases to the extent that $x^h_{p,t}$ adjusts from below to its long-run value $x^h_{p,*} = x^n_{p,t}$. The convergence of relative human capital stocks to their initial values is precisely the mechanism responsible for the decline in inequality. But again, it is important to stress the feasibility of the regime switch for the poor population group. Since the poor may start to invest in skills after the regime switch of the middle class has occurred, their threshold income may grow faster than their actual incomes. Thus, the poor never (or only temporarily) switch to the high-skill regime. In this case, their relative human capital shrinks toward zero and the initial distribution of wealth will affect long-run inequality. In this context, the overall amount of inequality may be a poor predictor for the feasibility of the regime switch for all social classes. What matters is the income gap between the poor and the middle class. A comparatively large distance between the poor and the middle class induces a relatively early switch of the middle class to the high-skill regime, which is responsible for a relatively fast growing threshold income of the poor. In addition, the role of the quality of the education sector reflected by κ is twofold. An increase in κ increases labor incomes and bequests, such that the decline in bequests after the regime switch will be reduced or even non-existent. Moreover, an increase in the productivity of the education sector may disconnect the poor from the growth process because of an increase in the growth rate of their threshold income. On the other hand, a reduction in the quality of the education sector reduces inequality during the transition because of a slow increase in labor incomes and bequests. But precisely the slower increase in incomes may again reduce the growth rate of incomes of the poor below the growth rate of their threshold incomes. Thus, for a given amount of inequality, there exists a certain range in which the productivity of the education sector ensures a regime switch for all social classes. We discuss and illustrate this arguments in the next section in more detail.

4 Numerical experiments

In this section we conduct numerical experiments to illustrate the theoretical results form the previous sections. Specifically, we analyze the effects of different amounts of initial inequality in wealth on the evolution of inequality over time.

We choose the parameters of the model such that the balanced growth path fits to empirical observations of developed economies. We fix the capital income share in the production of output, α, at 0.3. In our model, one period has a length of thirty years. The real interest rate is set to 4% per year, i.e., $R = 1.04^{30}$. The literature on business cycles suggests a discount factor of future consumption around 0.99 per quarter, such that $\beta = 0.99^{120}$. The long-run projections of the United Nations suggest a stationary world population, such that we set $n = 1$. As regards child-rearing time, we fix the time share necessary to raise one child to adulthood, z, at 0.027, which implies an opportunity cost of around 15% of parents' time endowment. Moreover, the weight of human capital in the parental utility function, γ, is set to 0.278.[14] The remaining values are calibrated in an iterative way assuring long-run human capital growth (per year) between one and two percent and expenditures on education not exceeding 6%, which implies $A = 10$, $\kappa = 3.3$, $\theta = 0.4$, $\bar{c} = 1.8$ and $\bar{e}_r = 0.28$, $\bar{e}_m = 0.27$, $\bar{e}_p = 0.26$.

In Figure 4, we depict the evolution of bequests [(a), (c), and (d)] and education [(b), (d), and (f)] in social class j for different amounts of initial inequality in the distribution of wealth (solid lines). In Figure 5 (solid lines), we depict the corresponding evolution of inequality as expressed by the income share of the rich and the evolution of relative human capital stocks governed by Equation (30). In both figures, the dashed lines show the hypothetical transition if the corresponding social class would not have switched to the high-skill regime. Scenario (a), (b) is characterized by a comparatively equal initial distribution of wealth, while the scenarios (c), (d) and (e), (f) exhibit not only a higher amount of initial inequality in wealth, but also different distances between the middle class and the poor. The latter will be crucial for the likelihood that the poor conduct a regime switch to the high-skill regime. Indeed, it may be misleading to assess the evolution of the economy just by means of initial inequality.

Since $\bar{e}_r > \bar{e}_m > \bar{e}_p$, social classes transit along different trajectories toward different steady states in the neoclassical regime (see Proposition 2 and Figure 2), in the sense that $b_{r,*}^n > b_{m,*}^n > b_{p,*}^n$. Moreover, Proposition 1 implies that the three social classes exhibit different threshold incomes: $\hat{I}_r > \hat{I}_m > \hat{I}_p$. Due to a higher level of assets, the rich reach their threshold level of income first and start to invest in education of their children. In the previous section, we argued that the regime switch to the high-skill regime induces a reduction in bequests (see Proposition 4 and Figure 3). In Figures 4 (a), (c), and (e) we see indeed that bequests of the rich fall short of the level in the neoclassical regime after the regime switch. Nevertheless, increasing investments in education increase labor incomes in subsequent periods such that the level of bequests increases again. Figures

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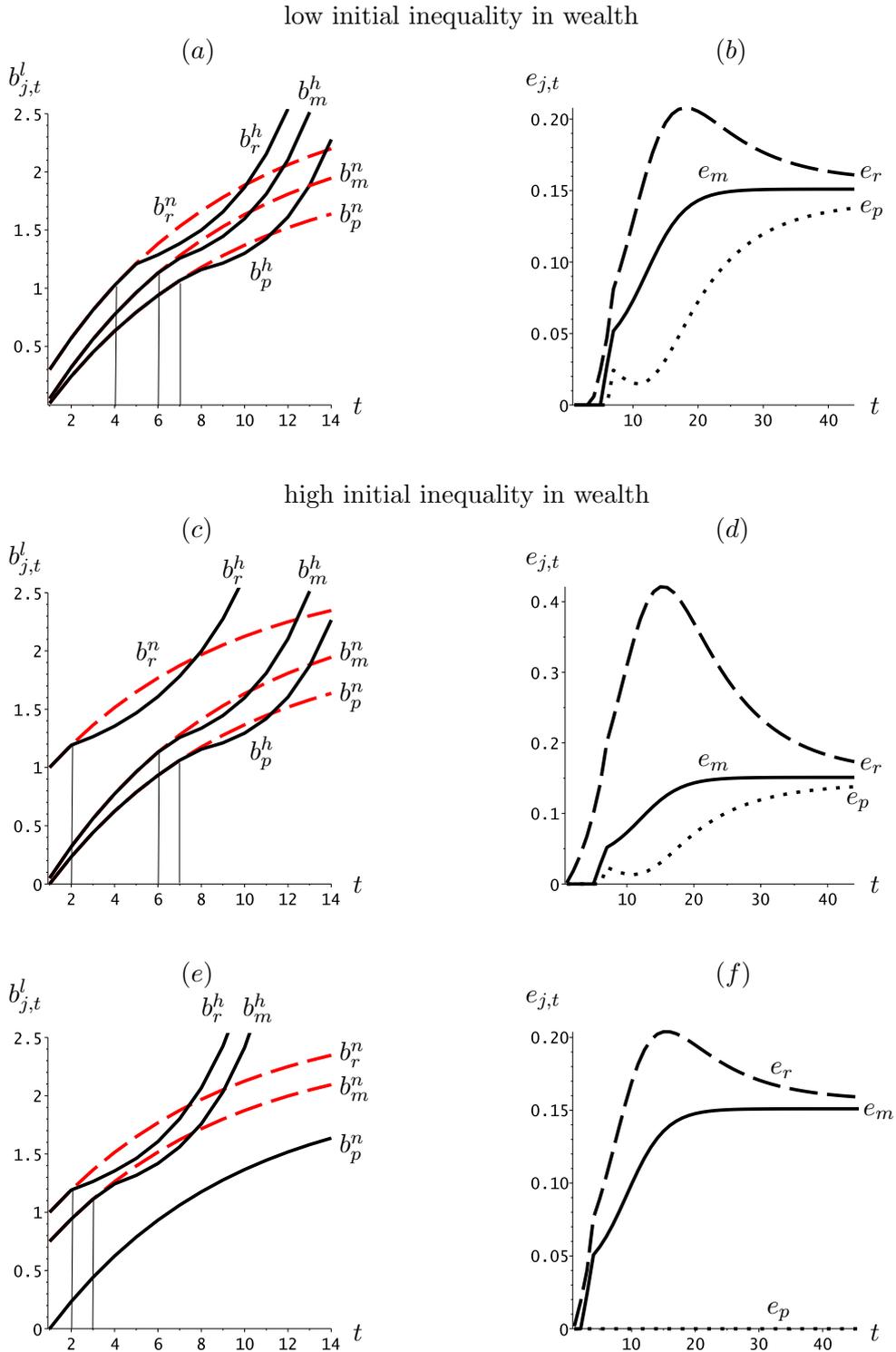[14]For further details on the evaluation of these values see de la Croix and Doepke (2003).

Figure 4: Evolution of bequests (b_j) and education (e_j): $(a), (b)$ low initial inequality; $(c), (d)$ high initial inequality; $(e), (f)$ high inequality between middle class and poor.
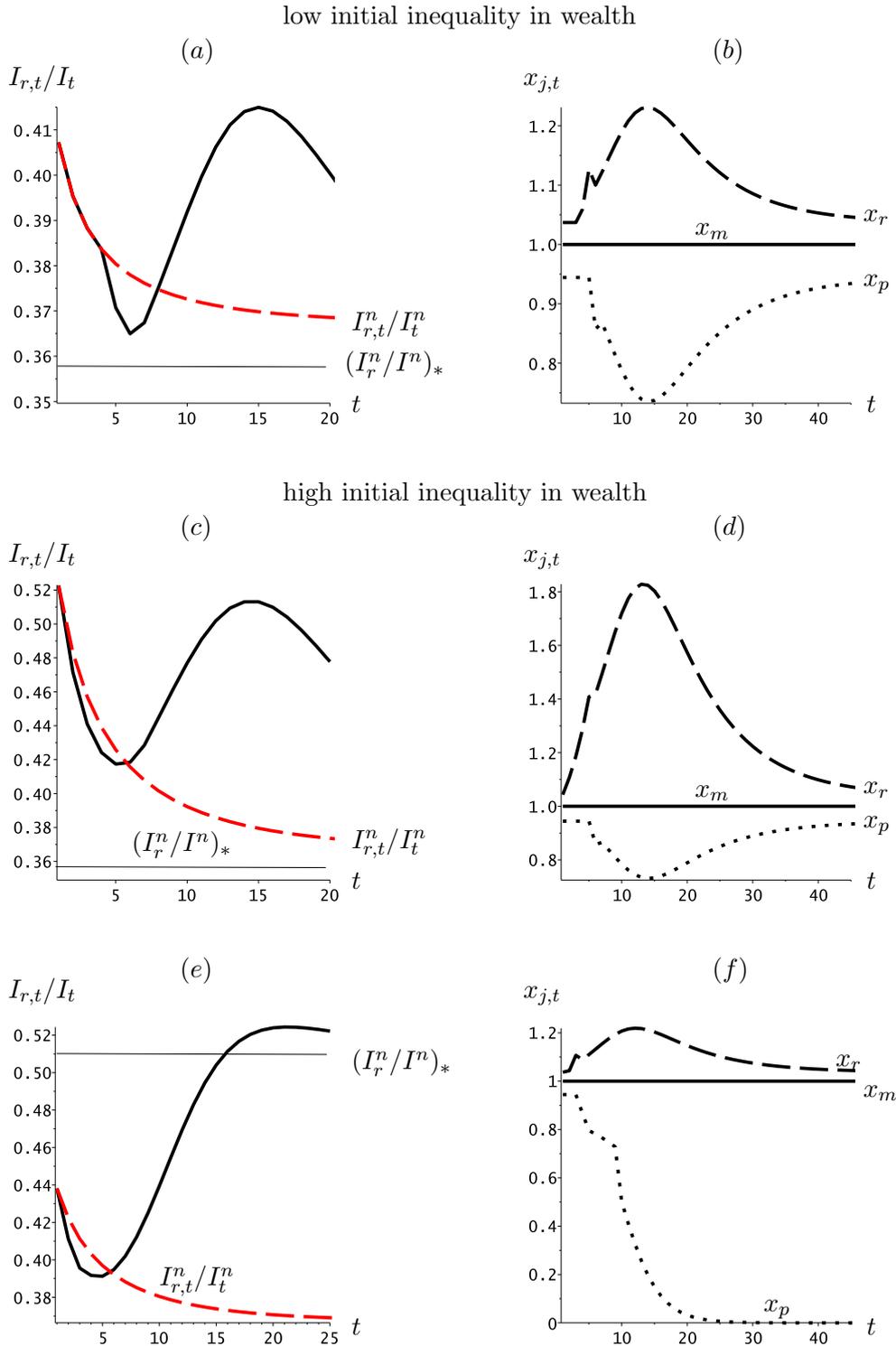
low initial inequality in wealth

(a)

$I_{r,t}/I_t$

(b)

$x_{j,t}$

high initial inequality in wealth

(c)

$I_{r,t}/I_t$

(d)

$x_{j,t}$

(e)

$I_{r,t}/I_t$

(f)

$x_{j,t}$

Figure 5: Incomes of the rich relative to total incomes $(I_{r,t}/I_t)$ and relative human capital endowments (x_j).

4 (b), (d) and (f) show the evolution of parental expenditures on education $(e_{j,t})$. The rich are the first to invest in eduction of their children, while the middle class and the poor exhibit constant levels of human capital until they experience a regime switch as well. Thus, human capital and incomes of the rich grow faster than in the other classes. Symmetrically, the poor are the last social group to begin to invest in education. At this moment the middle class and thus the education sector is characterized by growing levels of human capital, which in turn induces increasing costs for higher education. Hence, from the perspective of the poor, the regime switch goes hand in hand with shrinking bequests and increasing costs of higher education, such that the education levels of the poor undershoot. This is the mechanism responsible for the increase in inequality after the regime switch. Moreover, comparing Figures $4(a), (b)$ with $(c), (d)$ shows that initial inequality does not affect differences in education. However, comparing both scenarios with $(e), (f)$ demonstrates that the initial distribution affects the possibility of the regime switch of the poor, i.e., what matters is the distance between the level of wealth of the poor and those of the middle class. In the last scenario, inequality is initially even lower than in scenario $(c), (d)$, but the poor are comparatively poorer and accumulate wealth at a lower rate than the middle class due to subsistence consumption constraints. The middle class switches to the high skill regime, which induces a continuous increase in the poor's threshold income that exceeds the growth rate of their income. It follows that the poor never switch to the high skill regime.[15]

In light of Proposition 3 item (iii), relative human capital stocks converge to $\bar{e}_j/\bar{e}_m$ such that even the increase in inequality caused by the regime switch to the high-skill regime is just a transitory phenomenon and inequality will fall to a level determined by the differences in the levels of baseline education. We depict the evolution of inequality expressed by the dynamics of the share of total income of the rich in Figure 5. Initially, inequality is declining due to the forces of a neoclassical convergence mechanism.[16] After the regime switch to the high-skill regime, the income share of the rich drops below the level of the neoclassical regime because of a decline in bequests. It starts to increase again when the other social classes invest in education because their bequests are reduced, while labor incomes and bequests of the rich are increasing again. After investments in education of the rich have peaked, inequality starts to decline to its steady-state level. The evolution of inequality in later phases of the high-skill regime thus follows the evolution of relative human capital endowments (see right-hand panel of Figure 5).

Higher initial inequality due to a wealthier rich class amplifies the time interval during which the regime switches of the different social groups occurs such that the rich invest earlier in education and exhibit earlier increasing levels of bequests. The income share

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[15]For moderate increases in the poor's initial level of wealth, the poor may initially switch to the high-skill regime but since their income growth cannot keep up with the growth rate of their threshold income, $I'_{p,t}$ overtakes their incomes and the poor switch back to the neoclassical regime. In this case bequests converge from below to the neoclassical trajectory.

[16]Inequality shrinks if the rich are – compared to the other social classes – already close to their steady state. Otherwise inequality would increase due to the forces to cover subsistence needs.
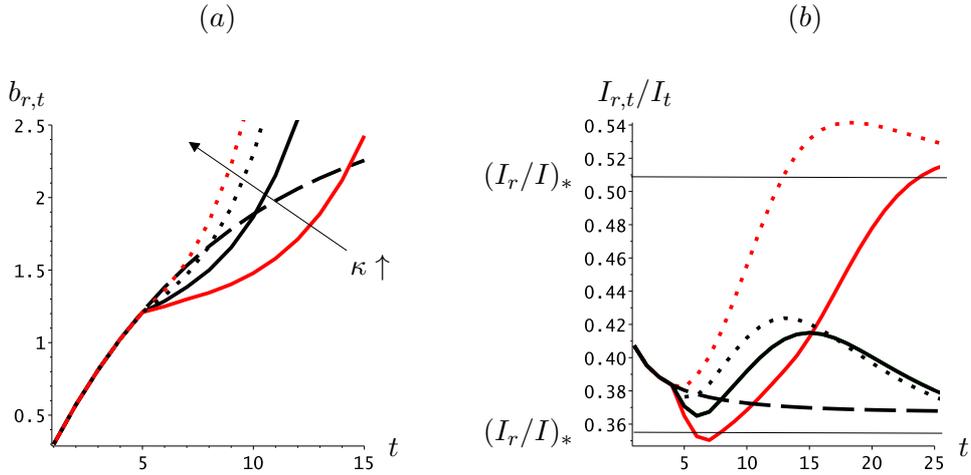
Figure 6: Evolution of (a) bequests for the rich (b_r) and (b) the income share of the rich $(I_{r,t}/I_t)$ in response to changes in the productivity of the education sector, κ; black solid: baseline calibration; red solid: κ reduced by 10%; black dotted: κ increased by 10%; red dotted: κ increased by 20% relative to the baseline calibration.

of the rich drops during the neoclassical regime from a higher level and increases due to the described differences in educational attainments to a higher level again. Nevertheless, differences in initial inequality do not affect the long-run income share of the rich. Long-run income shares are only affected if at least one group does not switch to the high-skill scenario. The long-run amount of inequality is then dependent on population shares of the different social classes. In this respect, the amount of initial inequality is, as has been stressed before, a poor predictor for long-run inequality. Indeed, scenario $(e),(f)$ is characterized by lower initial but a higher long-run inequality compared to scenario $(b),(c)$. What matters is not the overall amount of inequality but the distance between the middle class and the poor. Even though initial inequality may be comparatively low, the early switch of the middle class to the high-skill regime induces a fast growth rate of the poor's critical income level, such that their investments in education are equal to (or approach) zero, which gives rise to a larger amount of long-run inequality.

In Figure 6, we illustrate the effect of variations in the quality of the education sector, κ, on the evolution of inequality. Panel (a) presents the evolution of bequests for the rich and in panel (b) we depict the evolution of the income share of the rich. As becomes apparent, the productivity of the education sector steers the magnitude of the decline in inequality after the regime switch to the high-skill regime. The lower the productivity of the education sector, the longer the period of time during which agents reduce their bequests below the level of the neoclassical regime. Although it is a utility maximizing strategy to invest in higher education at the expense of bequests, a lower productivity of

the education sector implies a slower increase in wage incomes for subsequent generations. This contributes to a stronger transitory decline in inequality. A relatively low κ (solid red line) may be equally harmful for the poor as a relatively high κ. If κ is low, the reduction in bequests and the slow increase in labor incomes may create a situation that is characterized by a faster increase in the threshold income of the poor compared to their actual incomes, such that the poor will cease to invest in higher education. Hence, inequality rises after the regime switch to a high long-run value because the poor are disconnected from the high-skill regime. As regards the long-run effects, this scenario is symmetric to the one characterized by a relatively high productivity in the education sector (dotted red line). In the latter scenario, the poor's threshold income is increasing faster than their actual incomes right from the beginning, such that the poor would not even invest in higher education during the transition. Only for an intermediate range of κ the regime switch of all social classes is ensured. A moderate increase in the quality of the education sector (black dotted line) reduces the decline in inequality after the regime switch and increases the peak of inequality. however, after the peak inequality level is surpassed, inequality declines faster toward its long-run value.

5   Conclusions

We set up a novel overlapping generations model with three social classes: the rich, the middle class, and the poor. Initially, the economy is in the neoclassical regime in which income growth is entirely driven by the accumulation of physical capital. During this stage of economic development, investments in higher education are not widespread because higher education for the children is seen as a luxury good from the perspective of households.

After a certain threshold level of income is surpassed, richer households start to invest in higher education of their children. This, however, reduces their savings and therefore their bequests, which further reduces inequality for at least one generation. The corresponding earlier onset of higher education and therefore faster human capital accumulation of the children of the rich leads to an increase of their incomes. Subsequently, also the middle class and possibly the poor start to accumulate human capital. Since different social groups accumulate human capital at different rates, inequality starts to increase again. This mechanism gives rise to a candidate explanation of the observed U-shaped evolution of inequality, which we call the "Piketty curve". However, in contrast to Piketty, we argue that the top income share does not approach 1 in the long run. In the high-skill regime, there is long-run convergence of the growth rates of human capital between the rich, the middle class, and the poor, such that inequality declines again and the top income share converges toward a level substantially below 1. Crucial for the emergence of declining inequality, though, is that all social classes switch to the high-skill regime. Essentially, the poor face a risk of being disconnected from the growth process. Regarding the feasibility of a regime switch for the poor, the initial distribution of wealth is a poor predictor for the

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evolution of inequality. What matters is the distance between the poor and the middle class.

To focus on the higher education channel, we isolated it by deliberately abstracting from other mechanisms that affect inequality such as the disruptions of World War II, changing tax policies, declining population growth, skill-biased technological change, the decline of unionization, and globalization. We believe that each of these channels have contributed to the pattern of the development of inequality over the last century. Our aim was merely to emphasize and formalize the role of higher education. Analyzing the relative importance of the mentioned mechanisms for the evolution of inequality for different countries is a promising task for further research.

We also showed that it is by no means guaranteed that a regime switch from a neoclassical growth regime, in which income growth is driven by the accumulation of wealth, toward a high-skill growth regime, in which income growth is driven by human capital accumulation, takes place. A necessary condition for this to happen is that the threshold income level above which the rich start to accumulate human capital is below their steady-state income level in the neoclassical growth regime. If this is not the case, then a country might be stuck in a middle income trap from which it cannot escape toward the high-skill regime on its own. There is also an intermediate case in which one or more (but not all) classes are able to surpass the threshold income level above which it becomes optimal to invest in higher education.

A central policy implication of our framework to reduce inequality is to invest in public universities or in education subsidies for the poor (e.g., stipends). The reason is that such a policy reduces the costs of education for the corresponding social class and thereby reduces the threshold level of income above which it becomes optimal to invest in higher education. In general, a policy that raises education and thereby the human capital level of the population might also be a potential solution for a country to escape the middle income trap as indicated by the results of Eichengreen et al. (2013).

Finally, we want to mention that we abstracted from technological progress. Including an exogenously growing stock of technologies or even endogenous technological progress would leave, however, the substance of our framework unaffected because the threshold levels of income that are necessary for a regime switch would grow with the rate of technological progress.

Acknowledgments

Mathematical Appendix

A.1 Lemma 1

(1) Item (i):

If $l = n$, a member of social class j maximizes (5) subject to (8) and (9). The associated first-order conditions read

$$\frac{1}{c_{j,t}^n - \bar{c}} = \lambda, \tag{A.1}$$

$$\frac{\beta}{c_{j,t+1}^n} = \frac{\lambda}{R}, \tag{A.2}$$

$$\frac{\theta\beta}{b_{j,t+1}^n} = \frac{\lambda n}{R}. \tag{A.3}$$

Combining (A.1) with (A.2) and (A.1) with (A.3) yields

$$c_{j,t+1}^n = \beta R(c_{j,t}^n - \bar{c}), \tag{A.4}$$

$$b_{j,t+1}^n = \frac{\theta}{n} c_{j,t+1}^n. \tag{A.5}$$

Combining the last two expressions with the budget constraint gives item (i).

(2) Item (ii):

If $l = h$, agents maximize (5) subject to (8), (9), and (7). The associated first-order conditions are given by (A.1)-(A.3) for l=h and

$$\frac{\gamma}{e_{j,t}^h + \bar{e}_j} = wh_{m,t}^h n\lambda. \tag{A.6}$$

From the last expression and (A.1), we obtain

$$e_{j,t}^h = \frac{\gamma}{n}(c_{j,t}^h - \bar{c}) - \bar{e}_j. \tag{A.7}$$

Combining the last expression with (A.4) and (A.5) verifies item (ii).

A.2 Proposition 1

Noting (16), we obtain $e_{j,t}^h \leq 0$, if

$$\frac{\gamma I_{j,t}}{wh_{m,t}^h n} \leq [1 + (1 + \theta)\beta]\bar{e}_j + \frac{\bar{c}}{wh_{m,t}^h n}, \tag{A.8}$$

$$\Rightarrow I_{j,t}' = \frac{wh_{m,t}^h n\bar{e}_j[1 + (1 + \theta)\beta]}{\gamma} + \bar{c}. \tag{A.9}$$

A.3 Proposition 2

The regime switch requires $I_{j,*} = (1 - zn)w\bar{e}_j + Rb_{j,*} > I'_{j,t}$, which implies in light of Proposition 1 that

$$b^n_{j,*} \; > \; \frac{wh^h_{m,t}n\bar{e}_j}{\gamma R}[1 + (1 + \theta)\beta] + \frac{\bar{c}}{R} - \frac{(1 - zn)w\bar{e}_j}{R}. \tag{A.10}$$

Substituting now for $b^n_{j,*}$ by using (23) yields

$$\frac{\theta\beta R^2}{n[1 + (1 + \theta)\beta] - \theta\beta R^2}[(1 - zn)w\bar{e}_j - \bar{c}]$$
$$> \frac{wh^h_{m,t}n\bar{e}_j}{\gamma}[1 + (1 + \theta)\beta] + \bar{c} - (1 - zn)w\bar{e}_j. \tag{A.11}$$

From the last expression, we obtain a minimum level of $\hat{e}^l$ that assures a regime switch in the future

$$\hat{e}^l = \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - wh^l_{m,t}[n(1 + (1 + \theta)\beta) - \theta\beta R^2]\}}, \tag{A.12}$$

such that

$$\hat{e}^h = \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - wh^h_{m,t}[n(1 + (1 + \theta)\beta) - \theta\beta R^2]\}}, \tag{A.13}$$

and

$$\hat{e}^n = \frac{\gamma\bar{c}}{\{\gamma(1 - zn)w - w\bar{e}_m[n(1 + (1 + \theta)\beta) - \theta\beta R^2]\}}. \tag{A.14}$$

Obviously, $\hat{e}^n$ is constant and a regime switch is possible if $\bar{e}_j > \hat{e}^h$.

If $l = h$ it follows that $\hat{e}^h$ is growing with $h^h_{m,t}$, thus moving the critical level of income to the right, such that a regime switch of classes poorer than the middle class becomes infeasible if the threshold level has become greater than their steady-state level of bequests. Note also that $\hat{e}^n$ and $\hat{e}^h$ exhibit a vertical asymptote at

$$\tilde{h}_{m,t} = \frac{(1 - zn)\gamma}{n[1 + (1 + \theta)\beta] - \theta\beta R^2}. \tag{A.15}$$

A.4 Proposition 3

(i) The ratio between bequests and human capital is constant along the BGP.
Note that

$$\bar{e}_j + e_{j,t} = \frac{\gamma(I^h_{j,t} - \bar{c} + wh^h_{m,t}\bar{e}_j n)}{wh^h_{m,t}n[1 + \gamma + (1 + \theta)\beta]}. \tag{A.16}$$

Thus

$$\frac{h_{j,t+1}^h}{h_{j,t}^h} = (\bar{e}_j + e_{j,t})\kappa \frac{h_{m,t}^h}{h_{j,t}} = \frac{\gamma\kappa(I_{j,t}^h - \bar{c} + wh_{m,t}^h\bar{e}_j n)}{wh_{j,t}^h n[1 + \gamma + (1+\theta)\beta]}. \tag{A.17}$$

Note further that

$$\frac{b_{j,t+1}^h}{h_{j,t}^h} = \frac{\theta\beta R}{n[1 + \gamma + (1+\theta)\beta]}\left[\frac{I_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j}{h_{j,t}^h}\right] \tag{A.18}$$

$$\Rightarrow \frac{b_{j,t+1}^h}{h_{j,t+1}^h} = \frac{h_{j,t}^h}{h_{j,t+1}^h}\frac{\theta\beta R}{n[1 + \gamma + (1+\theta)\beta]}\left[\frac{I_{j,t}^h - \bar{c} + wh_{m,t}^h n\bar{e}_j}{h_{j,t}^h}\right]. \tag{A.19}$$

Combining the last expression with (A.17) yields

$$\frac{b_{j,t+1}^h}{h_{j,t}^h} = \frac{w\theta\beta R}{\gamma\kappa} = const. \tag{A.20}$$

(ii) The evolution of relative inequality. Noting that $x_{j,t}^h = h_{j,t}^h/h_{m,t}^h$, we obtain

$$x_{j,t+1}^h = \frac{\bar{e}_j + e_{j,t}}{\bar{e}_m + e_{m,t}} = \frac{\frac{I_{j,t}^h - \bar{c}}{wh_{m,t}^h} + \bar{e}_j n}{\frac{I_{m,t}^h - \bar{c}}{wh_{m,t}^h} + \bar{e}_m n} \tag{A.21}$$

$$\Rightarrow x_{j,t+1}^h = \frac{\left[(1 - zn) + \frac{Rb_{j,t}^h}{wh_{j,t}^h}\right]x_{j,t}^h - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_j n}{(1 - zn) + \frac{Rb_{m,t}^h}{wh_{m,t}^h} - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_m n}. \tag{A.22}$$

Taking into account (A.20), we obtain

$$x_{j,t+1}^h = \frac{\left[(1 - zn) + \frac{\theta\beta R^2}{\gamma}\right]x_{j,t}^h - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_j n}{(1 - zn) + \frac{\theta\beta R^2}{\gamma} - \frac{\bar{c}}{wh_{m,t}^h} + \bar{e}_m n}. \tag{A.23}$$

As $\lim_{t\to\infty} \frac{\bar{c}}{wh_{m,t}^h} = 0$, we obtain from the last expression that

$$x_{j,*} = x_{j,t+1} = x_{j,t} = 1 \tag{A.24}$$

if $\bar{e}_j = \bar{e}_m$ and

$$x_{j,*} = x_{j,t+1} = x_{j,t} = \frac{\bar{e}_j}{\bar{e}_m} \gtrless 1 \tag{A.25}$$

if $\bar{e}_j \gtrless \bar{e}_m$.

(iii) The gross growth rate of human capital.

From (A.17) and the definition of households' incomes, we obtain

$$\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa}{n[1+\gamma+(1+\theta)\beta]}\left[(1-zn)+\frac{Rb^h_{j,t}}{wh^h_{j,t}}-\frac{\bar{c}}{wh^h_{j,t}}+\frac{\bar{e}_j n}{x_{j,t}}\right]. \quad (A.26)$$

Combining the last expression with (A.20), we obtain

$$\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa}{n[1+\gamma+(1+\theta)\beta]}\left[(1-zn)+\frac{\theta\beta R^2}{\gamma}-\frac{\bar{c}}{wh^h_{j,t}}+\frac{\bar{e}_j n}{x_{j,t}}\right], \quad (A.27)$$

such that, in light of (ii), the gross growth rate of human capital reads

$$\lim_{t\to\infty}\frac{h^h_{j,t+1}}{h^h_{j,t}} = \frac{\gamma\kappa(1-zn+\bar{e}_m n)+\theta\beta R^2}{n[1+\gamma+(1+\theta)\beta]}. \quad (A.28)$$

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