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by
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Abstract

We introduce automation into the standard Solovian model of capital accumulation and show that (i) there is the possibility of perpetual growth, even in the absence of technological progress; (ii) the long-run economic growth rate declines with population growth, which is consistent with the available empirical evidence; (iii) there is a unique share of savings diverted to automation that maximizes the long-run growth rate of the economy; (iv) the labor share declines with automation to an extent that fits to the observed pattern.
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Jobs for every American is doomed to failure because of modern automation and production. We ought to recognize it and create an income-maintenance system so every single American has the dignity and the wherewithal for shelter, basic food, and medical care.
(Jerry Brown)

1 Introduction

Industrial robots, 3D printers, and intelligent devices based on machine learning have already taken over many (and will take over even more) of the tasks for which at least a small amount of labor input has been necessary up to now. For example, especially in the car industry, robots perform many production steps in an autonomous way[1]; 3D printers are capable of producing customized products with a minimal labor input (Abeliansky et al., 2015); driverless cars and lorries could soon transport goods from location A to location B without any involvement of labor; and devices based on machine learning are already able to diagnose some forms of diseases, to translate texts from one language to another with an acceptable quality, and even to write simple newsflashes (cf. The Economist, 2014; Lanchester, 2015; Brynjolfsson and McAfee, 2016). This implies that physical capital installed in the form of robots, 3D printers, driverless cars, and devices based on machine learning is close to being a perfect substitute for labor. By contrast, traditional physical capital installed in the form of machines, assembly lines, and factory buildings requires at least a certain amount of labor input to produce such that there is a complementarity between labor and traditional physical capital. Consequently, the more widespread adoption of automation has the potential to replace a large amount of workers in the production process without the compensating increase in labor demand that is usually concomitant to an expansion of production based on physical capital deepening. Most recently, Frey and Osborne (2013) have shown that 47 percent of total US employment is susceptible to job losses due to computerization and Autor and Dorn (2013) argue that there has already been a structural shift of labor supply from middle-income manufacturing to low-income services because the former is more susceptible of being replaced by automation than the latter.

We introduce automation into the standard framework of Solow (1956) and show that it has the potential to generate perpetual economic growth, even in the absence of technological progress. The reason is that automation capital resembles the properties of labor in the production process and the properties of traditional physical capital

[1] According to IFS (2015) there were 240,000 industrial robots sold worldwide in the year 2015 with a year-on-year growth of 8 percent from 2014. In terms of industries the demand was highest in the automotive industry followed by the electronics industry, while in terms of countries it was highest in China, Japan, the United States, the Republic of Korea, and Germany.

(to which we refer as "machines" from now on) in the accumulation process. In such a setting the decreasing returns to physical capital in a standard Cobb-Douglas production function are overcome because the accumulation of machines and automation capital *together* leads to constant returns with respect to the *overall* physical capital stock. This implies that, although all production factors have diminishing *marginal* returns, the aggregate production function exhibits constant returns with respect to the accumulable production factors.

This rather positive effect of automation comes with a downside, however. As we will see in the formal analysis, the properties of automation imply that its more widespread adoption reduces the wage rate because automation competes closely with the production factor labor. At the same time the income that is generated by automation is channeled toward the capital owners. Overall, the introduction of automation as it has been observed in the data between the 1970s and the 2010s implies a reduction of the aggregate labor income share by around 5.5 percentage points, which is in line with the observations reported by Karabarbounis and Neiman (2014). Considering the fact that capital income is typically much more unevenly distributed than labor income, this leads to an increase in inequality. Insofar, the widespread adoption of automation might face strong opposition from various parts of the population and the political spectrum, and not the least from labor unions.[2]

The main policy conclusion that emanates from our analysis is that it might be useful to design a compensation scheme to benefit the losers of automation technologies. Doing so could help to distribute the potentially enormous gains of automation more evenly among various parts of the society and thereby to ensure that automation technologies are adopted, while, at the same time, inequality would be kept in check. Without such a compensation scheme, a strong resistance to automation technologies can be expected, which has the potential to block their widespread adoption, at least for some time.

The literature on automation so far is scarce but there are a few exceptions. In a very interesting contribution, Steigum (2011) considers one aspect of automation, namely robots, as imperfect substitutes for labor in an optimal economic growth model. He shows that the possibility for perpetual economic growth exists and that the capital income share converges to 1 in this setting. However, he does not determine the growth-maximizing investment rate in terms of robots, population growth does not affect long-run economic growth in his framework, and the implications of the introduction

[2]This opposition would arguably be more justified than the opposition to the industrialization of the 19th Century. The reason is that there has always been a certain amount of complementarity between physical capital and labor during the industrialization process. This complementarity implies that, although there can be negative short-run effects of capital accumulation for some workers who are displaced, the overall long-run effect of capital accumulation is positive because the expansion of production requires more labor input, such that labor incomes rise with capital deepening. If there is a perfect substitutability between automation and labor, the positive long-run effects of capital accumulation on wages might be overturned.

of robots on the labor share are not analyzed quantitatively. Furthermore, his analysis is focused on the long-run growth rate in the sense that the transitional dynamics are not illustrated. We focus particularly on the impact of population growth on long-run growth, determine the growth-maximizing investment rate in terms of automation, quantify the potential decrease in the labor share due to automation, and illustrate the transitional dynamics of such a framework. In so doing we hope to shed some additional light on the impact of automation on modern economic development.

In an empirical evaluation, Graetz and Michaels (2015) estimate that the intensification of the use of industrial robots boosted growth of per capita GDP by 0.37 percentage points between 1993 and 2007. In addition, they find that the labor share of income is negatively related to an increase in the use of industrial robots, albeit the result is not statistically significant. Overall, these findings are consistent with the results of Steigum (2011) and with our theoretical implications.

The paper is structured as follows. In Section 2 we introduce automation into the standard Solow (1956) model. We show that this creates perpetual growth and allows us to analyze the long-run economic growth effects of a changing savings rate, of a changing population growth rate, and of a changing fraction of investment that is diverted to automation. Finally, we show that the introduction of automation could be responsible for the decline in the labor share that has been observed in most developed countries over the last decades. Section 3 contains some numerical experiments that illustrate our theoretical results. In Section 4 we conclude and discuss the potential implications of automation for economic policy.

2 The model

2.1 Basic assumptions

Consider an economy with three production factors, labor, physical capital installed in the form of machines, and automation capital. Time evolves continuously and is denoted by t. Following the literature, we abstract from unemployment and retirement such that the labor force grows with the rate of population growth n. Machines and automation capital are accumulable production factors that increase due to deliberate investments and decrease because of depreciation at the rate δ. Labor and machines are imperfect substitutes, as in the standard Solow (1956) model. Automation, by contrast, is a perfect substitute for the production factor labor and an imperfect substitute for the production factor machines. In other words, automation shares the properties of physical capital in the accumulation process, while it resembles the standard properties of labor in the production process.

There is a continuum of firms with each of them having access to a Cobb-Douglas production function of the form

$$Y(t) = A(t)[L(t) + P(t)]^{1-\alpha} K(t)^{\alpha}, \tag{1}$$

where $Y(t)$ is aggregate output, $L(t)$ refers to labor, $K(t)$ denotes physical capital installed in the form of machines, $P(t)$ denotes the stock of automation capital, and $A(t) \equiv 1$ refers to the level of technology, which we deliberately normalize to 1. Allowing for technological progress would not change the substance of our analysis but would obscure the main mechanisms that we aim to highlight. Due to perfect competition, production factors are employed up to the point at which they earn their marginal value product. Consequently, the factor rewards are given by

$$w(t) = (1-\alpha)\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha}, \qquad r(t) = R(t) - \delta = \alpha\left[\frac{L(t)+P(t)}{K(t)}\right]^{1-\alpha} - \delta, \tag{2}$$

where $w(t)$ is the wage rate, $r(t)$ is the interest rate, and where we note that automation is rewarded with $w(t) - \delta$. Obviously, the wage rate decreases with the number of workers *and* with automation, while it increases with the stock of machines. The converse holds true for the interest rate. The reason for the negative effect of automation on the wage rate is the perfect substitutability between automation and labor.

The economy is closed and we abstract from a government such that output is used for consumption $C(t)$ and savings $S(t)$ according to $Y(t) = C(t) + S(t)$. As is well-known in such a setting, savings are equal to investment $I(t)$ such that $I(t) = S(t) = sY(t)$, where s is the exogenous constant savings rate, i.e., the fraction of gross income that households set aside for future consumption. In contrast to the standard Solow (1956) model, investments can be made in terms of two different forms of capital: machines and automation. For simplicity, we assume that a share s_m of savings is diverted to investment in terms of machines and a share $1 - s_m$ is diverted to investment in terms of automation. Altogether, this setup yields the following accumulation equations for machines and automation, respectively:

$$\dot{K}(t) = s_m I(t) - \delta K(t), \qquad \dot{P}(t) = (1-s_m)I(t) - \delta P(t). \tag{3}$$

Using the production function (1), the growth rates of machines and of automation can be written as, respectively,

$$\frac{\dot{K}(t)}{K(t)} = s_m s\left[\frac{K(t)}{L(t)+P(t)}\right]^{-(1-\alpha)} - \delta, \tag{4}$$

$$\frac{\dot{P}(t)}{P(t)} = (1-s_m)\, s\frac{1+[P(t)/L(t)]}{P(t)/L(t)}\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha} - \delta. \tag{5}$$

Output per worker is given by

$$y(t) = \frac{Y(t)}{L(t)} = [1 + p(t)]^{1-\alpha} k(t)^{\alpha}, \tag{6}$$

where lowercase letters refer to variables in terms of per worker units, i.e., for any variable $X(t)$ we have that $x(t) = X(t)/L(t)$. Reformulating the machine accumulation equation and the automation accumulation equation in per-capita terms yields

$$\begin{aligned}
\frac{\dot{K}(t)}{L(t)} &= s_m s[1 + p(t)]^{1-\alpha} k(t)^{\alpha} - \delta \frac{K(t)}{L(t)}, \\
\frac{\dot{P}(t)}{L(t)} &= (1 - s_m) s[1 + p(t)]^{1-\alpha} k(t)^{\alpha} - \delta \frac{P(t)}{L(t)}.
\end{aligned}$$

Both types of capital accumulate faster if the savings rate is higher and the rate of depreciation is lower. If the share of investment diverted to machines (s_m) is higher, then the accumulation of machines speeds up and the accumulation of automation capital slows down. In the Appendix we show that the following system of equations for the accumulation of machines per worker and automation capital per worker fully describes the dynamic evolution of the model economy

$$\begin{aligned}
\dot{k}(t) &= s_m s[1 + p(t)]^{1-\alpha} k(t)^{\alpha} - \delta \frac{K(t)}{L(t)} - nk(t), \\
\dot{p}(t) &= (1 - s_m) s[1 + p(t)]^{1-\alpha} k(t)^{\alpha} - \delta \frac{P(t)}{L(t)} - np(t),
\end{aligned}$$

where we see that population growth implies dilution in terms of both types of capital. Furthermore, we show that the economy converges to a situation in which the fraction of automation capital to machines is given by

$$\xi := p(t)/k(t) = \frac{1 - s_m}{s_m}.$$

Obviously, and as expected, $\xi = p(t)/k(t)$ declines with the fraction of investment that is diverted to machines (s_m) because an increase in s_m implies that relatively more machines are accumulated and therefore relatively less automation capital. At the steady state this translates into a higher machine intensity and a lower automation intensity. Finally, we show that the economy converges to a situation in which machines per worker, automation capital per worker, and GDP per worker all grow at the common constant rate

$$g = s \cdot s_m^{\alpha} (1 - s_m)^{1-\alpha} - \delta - n. \tag{7}$$

Note that, if the first term on the right hand side is large (e.g., because of a large enough savings rate), this growth rate is positive such that the model generates perpetual increases in income/production. In the Appendix we show that, if the savings rate is too low to generate positive long-run growth, then the economy converges to a steady state at which per capita variables do not grow, while aggregate variables grow at the rate of population growth. From now on we focus on the solution for which Equation (7) is positive. As we will see in the numerical section, using standard parameter values for the savings rate (s), the elasticity of final output with respect to machines (α), the depreciation rate (δ), and the population growth rate (n) indeed implies a positive right hand side of Equation (7). Altogether, this affords the following proposition, which is the first result of our paper.

Proposition 1. *If automation is considered as a perfect substitute for labor in the production function of a standard Solow (1956) model, then such a framework is able to generate perpetual economic growth, even in the absence of technological progress.*

This finding contrasts with the standard neoclassical growth model without technological progress, in which the rate of long-run growth is equal to zero. The reason for perpetual growth in our case is that the properties of automation capital in the production process, i.e., that automation is a perfect substitute for labor, help to overcome the diminishing marginal product of physical capital installed in the form of machines. The result of Proposition 1 is fully consistent with the result of Steigum (2011) that the introduction of robots into an optimal growth model implies positive long-run economic growth for some parameter values. In addition, it is also consistent with the empirical result of Graetz and Michaels (2015) that an intensification of the use of industrial robots boosts growth of productivity and of per capita GDP.

A standard approach to generate long-run growth in the neoclassical growth literature is to assume that technology improves at an exogenously given constant rate (Barro and Sala-i-Martin, 2004; Acemoglu, 2009). However, while the long-run growth rate is indeed positive in such a setting, it is still independent of the structural parameters of the model. Consequently, such a framework implies that changes in the savings rate and changes in the population growth rate have no impact on long-run economic growth whatsoever. As mentioned above, this is inconsistent with the empirical literature on the determinants of long-run economic growth (see also Durlauf et al., 2005, for an extensive overview). As is obvious from Equation (7), our framework, by contrast, implies a dependence of economic growth on the savings rate and on population growth. Inspecting the growth rate as expressed in Equation (7), we can state the following proposition.

Proposition 2. *In the Solow (1956) model with automation, the long-run economic growth rate increases with savings/investments and decreases with the rate of population growth.*

The intuition behind this finding is straightforward. Since the diminishing marginal product of physical capital in the standard model is overcome by the use of automation capital, variables that raise the overall accumulation rate of physical capital also raise the long-run economic growth rate. The converse holds true for variables that reduce the overall accumulation rate of physical capital such as capital dilution due to population growth. This result is consistent with the available empirical evidence as far as the positive correlation between investment and growth is concerned (see, for example, Barro, 1991, 1997; Sala-i-Martin, 1997) and as far as the negative correlation between population growth and economic growth – that is found for developed countries throughout the 20th Century – is concerned (see, for example, Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Li and Zhang, 2007; Herzer et al., 2012). The positive growth effect of savings would also be implied by endogenous growth models of the Romer (1990) type and by the framework of Steigum (2011), while the latter effect is difficult to generate in endogenous growth models and it is not analyzed explicitly in Steigum (2011).

The result of a negative relationship between economic growth and population growth is also interesting from the point of view of semi-endogenous and scale-free Schumpeterian growth models that often counterfactually imply a weak scale effect, i.e., that countries with faster population growth feature faster long-run economic growth. The weak scale effect has often been eliminated by the detailed modeling of the accumulation of human capital in the sense that faster population growth implies that fewer resources are available to invest in the education of children (cf. Dalgaard and Kreiner, 2001; Strulik, 2005; Bucci, 2008; Strulik et al., 2013; Boikos et al., 2013; Bucci, 2013; Prettner, 2014). In such a setting, the quality-quantity tradeoff at the micro level translates into a negative (or in some models a non-monotonic) relationship between population growth and economic growth at the macro level. Our framework shows that – at a stage of development at which automation is adopted – there exists a complementary mechanism of sustained capital dilution due to population growth that reinforces the channel via the quality-quantity tradeoff.

Next we turn our attention to the fact that there is a unique fraction of investment diverted to automation that maximizes the long-run growth rate of the economy. In this regard, we are able to state the following proposition.

Proposition 3. *In the Solow (1956) model with automation, the growth rate of the economy increases with the share of savings that is used for automation (machines) as long as the fraction of savings diverted to machines is larger (smaller) than the elasticity of output with respect to machines.*

Proof. For the proof we calculate the derivative of g with respect to s_m:

$$\frac{\partial g}{\partial s_m} = s \cdot s_m^{\alpha-1}(1-s_m)^{-\alpha}(\alpha - s_m).$$

We see that this expression is positive if $\alpha - s_m$ is positive and it is negative if $\alpha - s_m$ is negative. $\square$

The intuition for this result is the following. A reduction in the share of gross investment diverted to machines would lead, ceteris paribus, to a *reduction* in economic growth. However, the reduction in the share of gross investments diverted to machines comes with a corresponding increase in the share of gross investments diverted to automation. The latter would, ceteris paribus, lead to an *increase* in economic growth. If the fraction of gross investments diverted to machines is larger (smaller) than the elasticity of final output with respect to machines in the production function, the reduction in growth due to a lower accumulation rate of machines is smaller (larger) than the corresponding increase in the rate of economic growth due to an increase in the accumulation rate of automation. Consequently, economic growth is maximized if $s_m = \alpha$.

Now we turn our attention to the implication that the introduction and initial adoption of automation has on the labor income share of an economy. In our case, aggregate labor income is given by

$$w(t)L(t) = (1-\alpha)\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha} L(t), \tag{8}$$

which implies that the labor income share pins down to

$$\frac{w(t)L(t)}{Y(t)} = (1-\alpha)\frac{L(t)}{L(t)+P(t)}. \tag{9}$$

We immediately see that the accumulation of automation capital reduces the labor income share in such a setting and summarize this finding in the following proposition.

Proposition 4. *If we consider automation in a standard Solow (1956) model, an increase in the stock of robots reduces the labor income share of the economy.*

The intuition for this finding is the following. From the production technology it is obvious that the wage rate decreases and the capital rental rate increases, if, ceteris paribus, the stock of automation capital increases. Since the income that is generated by automation is used to compensate capital owners, the increase in automation implies that the capital income share rises and the labor income share declines. To put it differently, automation competes with labor and therefore its widespread adoption reduces wages, while, at the same time, the income that automation generates is channeled to the capital owners. Therefore, our framework proposes a complementary way of explaining the empirical finding of a decreasing labor income share in most developed countries over the last decades (see, for example, Elsby et al., 2013; Schmidt and Vosen, 2013; Karabarbounis and Neiman, 2014; Piketty, 2014, for a discussion and for complementary channels). As far as the empirical relevance of this finding is

Table 1: Parameter values for the numerical analysis

Parameter	Value	Comment
s	0.21	Average gross investment rate (2000-2013) for the US
s_m	0.7	Arbitrary value
α	0.3	Jones (1995), Acemoglu (2009), Grossmann et al. (2013)
δ	0.04	Grossmann et al. (2013)
n	0.009	Average rate (2000-2014) for the US (World Bank, 2015)
$L(0)$	1	Arbitrary initial value
$K(0)$	1	Arbitrary initial value
$P(0)$	1	Arbitrary initial value

concerned, Graetz and Michaels (2015) indeed find a negative correlation between the intensification of the use of industrial robots and the labor share. However, their result is, while being of a large magnitude, not statistically significant.

3 Numerical illustration

In this section we illustrate the trajectories that are implied by our model for parameter values that are either taken from the literature or that are implied by the data for the United States (cf. World Bank, 2015). We set the gross savings rate s equal to the average gross domestic investment rate over the years 2000 to 2013 and the population growth rate n equal to the geometric average of the population growth rate over the years 2000 to 2013. Furthermore, we use a value of 0.3 for the elasticity of final output with respect to physical capital (α), which is in line with the literature (cf. Jones, 1995; Acemoglu, 2009; Grossmann et al., 2013). Finally, we set the rate of depreciation equal to $\delta = 0.04$ as in Grossmann et al. (2013). Table 1 summarizes the parameter values that we use for our numerical illustration and provides a short justification for them. Irrespective of the fact that we use "realistic" parameter values, we do not claim to calibrate the model to the data because our aim is merely to illustrate the effect that automation can have on economic growth. In so doing we abstract from important other factors that determine long-run economic growth such as technological progress and human capital accumulation (cf. Romer, 1990; Lucas, 1988; Strulik et al., 2013). Without these elements, the quantitative description of long-run growth is, for sure, incomplete. Claiming that our framework is able to generate the observable growth trajectories is therefore not justified.[3]

In Figure 1 we plot, on the left side, the physical capital stock per capita, the automation capital stock per capita, and per capita GDP against time from $t = 0$ to

[3] A model formulation that includes human capital, purposeful R&D, and automation together is beyond the scope of the present paper (in which we rather aim to show the effects that automation can have on economic growth) but it is surely a promising avenue for further research to address the quantitative implications of automation for economic growth.

$t = 100$. On the right side we plot the corresponding growth rates. The solid lines refer to the baseline parameter specification displayed in Table 1. We clearly observe exponential growth in the physical capital stock, the stock of automation capital, and per capita GDP with no tendency to level off in the long run. Furthermore, we see that the growth rates of these variables converge toward their long-run solutions that are clearly positive. This numerical example illustrates the result of Proposition 1.

Furthermore, we illustrate what happens if the gross savings rate increases from the baseline level of $s = 0.21$ to a rate of $s = 0.25$. While the solid lines represent the original solution, the dashed lines represent the new solution with the higher savings rate. We observe that the country with the higher savings rate grows faster, even in the long run. This is exactly what the first part of Proposition 2 implies and what the empirical growth literature establishes in terms of investment (cf. Barro, 1991, 1997; Sala-i-Martin, 1997; Sala-i-Martin et al., 2004).

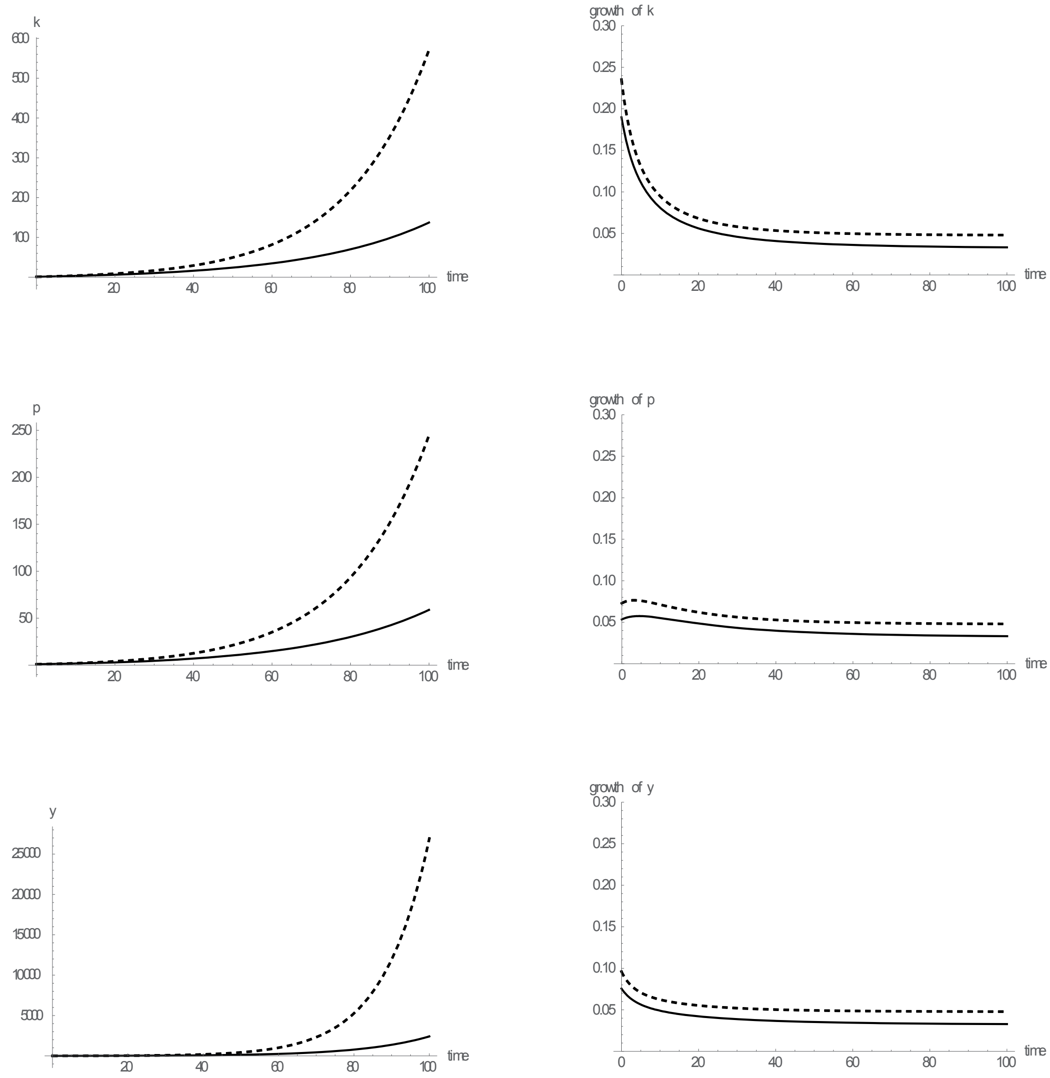

Figure 1: Levels of k, p, and y (left side) and growth rates of k, p, and y (right side). The solid lines represent the original solution, while the dashed lines represent the solution with the higher savings rate.

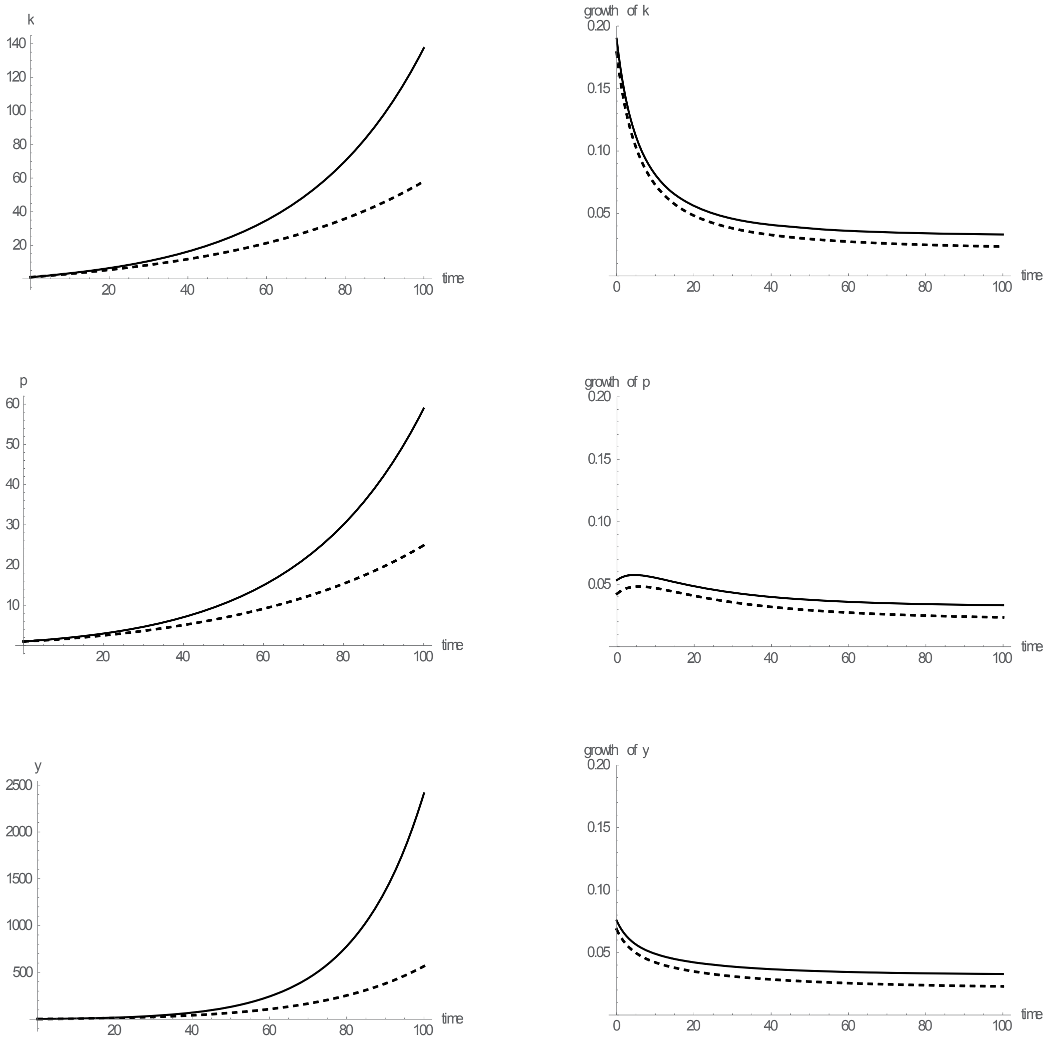

Figure 2: Levels of k, p, and y (left side) and growth rates of k, p, and y (right side). The solid lines represent the original solution, while the dashed lines represent the solution with the higher population growth rate.

In Figure 2 we show the impact of an increase in the population growth rate of $n = 0.009$ to a rate of $n = 0.02$. Again we plot, on the left side, the physical capital stock per capita, the stock of automation capital per capita, and per capita GDP against time and the corresponding growth rates on the right side. The solid lines represent the baseline solution, while the dashed lines represent the solution with the higher population growth rate. We observe that the country with the higher population growth rate attains a lower growth rate of per capita GDP, even in the long run. This is what the second part of Proposition 2 implies and what the empirical literature on the connection between economic growth and population growth establishes (Brander and Dowrick, 1994; Kelley and Schmidt, 1995; Ahituv, 2001; Li and Zhang, 2007; Herzer et al., 2012).

Finally, we assess the implied impact of the introduction of automation on the labor share. Karabarbounis and Neiman (2014) document a reduction of the global labor share by around 5 percentage points from the early 1970s to the 2010s. Given that the fraction of industrial robots to the total capital stock in advanced economies is estimated to be 2.25 percent in 2007 according to Graetz and Michaels (2015), and assuming that it was close to zero in the beginning of the 1970s, our framework would imply a decline of the labor share by around 5.5 percentage points, which is roughly in line with the data.

4 Conclusions

We introduced automation into the original model of Solow (1956). While the stock of automation capital is accumulated in a similar vein as physical capital installed in the form of machines, its properties in the production process resemble those of labor. We show that, in such a setting, there is perpetual growth of per capita output, even in the absence of technological progress. Furthermore, the long-run economic growth rate increases with the savings rate and declines with population growth, which is consistent with the available empirical evidence for developed countries in the 20th Century. Finally, we show that there is a unique share of savings diverted to the accumulation of automation capital that maximizes the long-run growth rate of the economy. Our framework has the potential to explain the decrease in the labor income share that has been observed in developed countries over the past decades. The reason is that automation competes closely with the production factor labor, while, at the same time, the income that automation generates is channeled toward the capital owners.

We deliberately abstracted from a number of features that a more realistic model would need to capture such as technological progress, human capital accumulation, the skill-based heterogeneity of the workforce, and the imperfect substitutability between skilled workers and automation. We do not think that the main results would change:

Adding technological progress and human capital accumulation would merely introduce other sources of economic growth, apart from capital accumulation, such that the results of Propositions 1, 2, and 3 would still hold. However, the imperfect substitutability between skilled workers and automation in an environment with skill-based heterogeneity of the workforce might attenuate the impact of automation on the labor share of aggregate income. Overall it is expected that automation would induce labor to shift to sectors in which a substitution between automation capital and workers is more difficult (cf. Autor and Dorn, 2013).

The main policy conclusion derives from the fact that automation has the potential to raise overall living standards substantially, while, at the same time, workers could be adversely affected. In particular, workers who perform the tasks that can be easily done by industrial robots, driverless vehicles, and intelligent devices based on machine learning might come under severe pressure. As a consequence, inequality could be expected to increase. To mitigate the increase in inequality and thereby to reduce the anticipated opposition to automation from labor unions, it might be desirable to set up a compensation scheme that is used to support the losers of automation technologies. That said, especially in economies that are aging rapidly and in which the labor force has already started to shrink, automation could be (part of) a solution to overcome the problems that are induced by the associated scarcity of labor.

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Appendix

A Derivations

A.1 Derivation of Equation (5)

Using Equation (3), together with the production function (1), we get

$$\frac{\dot{P}(t)}{P(t)} = (1 - s_m)s[L(t) + P(t)]^{1-\alpha} K(t)^{\alpha} P(t)^{-1} - \delta.$$

Multiplying the first term on the right hand side by $\{[L(t) + P(t)]/[L(t) + P(t)]\}^{\alpha}$ yields

$$\frac{\dot{P}(t)}{P(t)} = (1 - s_m)s \left[\frac{1 + P(t)/L(t)}{P(t)/L(t)}\right] \left[\frac{K(t)}{L(t) + P(t)}\right]^{\alpha} - \delta.$$

A.2 Derivation of the long-run accumulation rate of machines and automation

Reformulating the machine accumulation equation in per-capita terms yields

$$\frac{\dot{K}(t)}{L(t)} = s_m s[1+p(t)]^{1-\alpha} k(t)^\alpha - \delta \frac{K(t)}{L(t)}.$$

Reformulating the automation accumulation equation in per-capita terms yields

$$\frac{\dot{P}(t)}{L(t)} = (1-s_m)s[1+p(t)]^{1-\alpha} k(t)^\alpha - \delta \frac{P(t)}{L(t)}.$$

The dynamics of $k(t)$ and $p(t)$ are then given by

$$\begin{aligned}
\dot{k}(t) &= \frac{d\frac{K(t)}{L(t)}}{dt} = \frac{\dot{K}(t)}{L(t)} - \frac{K(t)}{L(t)^2}\dot{L}(t) = \frac{\dot{K}(t)}{L(t)} - k(t)\frac{\dot{L}(t)}{L(t)} = \frac{\dot{K}(t)}{L(t)} - nk(t), \\
\dot{p}(t) &= \frac{d\frac{P(t)}{L(t)}}{dt} = \frac{\dot{P}(t)}{L(t)} - \frac{P(t)}{L(t)^2}\dot{L}(t) = \frac{\dot{P}(t)}{L(t)} - p(t)\frac{\dot{L}(t)}{L(t)} = \frac{\dot{P}(t)}{L(t)} - np(t).
\end{aligned}$$

Taken together, these results imply the following system of equations for the evolution of machines per worker and automation capital per worker

$$\begin{aligned}
\dot{k}(t) &= s_m s[1+p(t)]^{1-\alpha} k(t)^\alpha - \delta \frac{K(t)}{L(t)} - nk(t), \\
\dot{p}(t) &= (1-s_m)s[1+p(t)]^{1-\alpha} k(t)^\alpha - \delta \frac{P(t)}{L(t)} - np(t),
\end{aligned}$$

In terms of growth rates we have

$$\begin{aligned}
\frac{\dot{k}(t)}{k(t)} &= s_m s \left[\frac{1+p(t)}{k(t)}\right]^{1-\alpha} - \delta - n, \\
\frac{\dot{p}(t)}{p(t)} &= (1-s_m)s \left[\frac{1+p(t)}{p(t)}\right]^{1-\alpha} \left[\frac{k(t)}{p(t)}\right]^{\alpha} - \delta - n.
\end{aligned}$$

Now we denote the growth rate of a variable x by g_x and the growth rate of its growth rate by g_{g_x}. Then we have

$$\begin{aligned}
g_k &= s_m s \left[\frac{1+p(t)}{k(t)}\right]^{1-\alpha} - \delta - n, && (10)\\
g_p &= (1-s_m)s\left[\frac{1+p(t)}{p(t)}\right]^{1-\alpha}\left[\frac{k(t)}{p(t)}\right]^{\alpha} - \delta - n, && (11)\\
\Rightarrow \log(g_k) &= \log(s_m) + \log(s) + (1-\alpha)\log[1+p(t)] \\
&\quad -(1-\alpha)\log[k(t)] - \log(\delta - n), && (12)\\
\Rightarrow \log(g_p) &= \log(1-s_m) + \log(s) + (1-\alpha)\log[1+p(t)] - (1-\alpha)\log[p(t)] \\
&\quad +\alpha\log[k(t)] - \alpha\log[p(t)] - \log(\delta - n), && (13)\\
\Rightarrow g_{g_k} &= (1-\alpha)\frac{\dot{p}(t)}{1+p(t)} - (1-\alpha)g_k, && (14)\\
\Rightarrow g_{g_p} &= (1-\alpha)\frac{\dot{p}(t)}{1+p(t)} - (1-\alpha)g_p + \alpha g_k - \alpha g_p. && (15)
\end{aligned}$$

Since, at the long-run equilibrium [for large $p(t)$], we have that

$$\frac{\dot{p}(t)}{1+p(t)} \approx g_r,$$

Equations (14) and (15) imply that the economy converges to a long-run growth rate with $g_p \approx g_k \equiv g$. Note that, for large $p(t)$ and large $k(t)$, we have

$$\left[\frac{1+p(t)}{p(t)}\right]^{1-\alpha} \approx 1, \qquad \frac{p(t)}{k(t)} \approx \frac{1+p(t)}{k(t)} := \xi.$$

Then we can rewrite Equations (10) and (11) such that

$$\begin{aligned}
g &= s_m s \xi^{1-\alpha} - \delta - n, && (16)\\
g &= (1-s_m)s\left[\frac{1}{\xi}\right]^{\alpha} - \delta - n. && (17)
\end{aligned}$$

These are two equations in the two unknowns g and ξ. Equalizing their right hand sides yields

$$\begin{aligned}
(1-s_m)s\left[\frac{1}{\xi}\right]^{\alpha} &= s_m s \xi^{1-\alpha},\\
\frac{1-s_m}{s_m} &= \xi.
\end{aligned}$$

Obviously, and as expected, $\xi = p(t)/k(t)$ declines in s_m because an increase in s_m means that relatively more machines are accumulated and relatively less automation capital. Plugging (18) into (16) yields the long-run growth rate of the economy as

$$g = s_m \cdot s \left(\frac{1 - s_m}{s_m} \right)^{1-\alpha} - \delta - n = s s_m^{\alpha} (1 - s_m)^{1-\alpha} - \delta - n.$$

A.3 The long-run growth rate of GDP per worker

Since output per worker is given by

$$y(t) = \frac{Y(t)}{L(t)} = [1 + p(t)]^{1-\alpha} k(t)^{\alpha},$$

we have that

$$\log[y(t)] = (1 - \alpha) \log[1 + p(t)] + \alpha \log[k(t)]$$

such that

$$\frac{\dot{y(t)}}{y(t)} = (1 - \alpha) \frac{\dot{p(t)}}{[1 + p(t)]} + \alpha \frac{\dot{k}}{k}.$$

Using

$$\frac{\dot{p}(t)}{1 + p(t)} \approx g_p$$

this implies

$$\frac{\dot{y}(t)}{y(t)} \approx g.$$

A.4 The steady state with stagnation

We have seen that our model is able to generate perpetual economic growth, even in the absence of technological progress. In the following we show that there is another solution that becomes only relevant for parameter settings in which Equation (7) would be zero or even negative, which we ruled out by assumption in the main part of the paper. The steady state for which Equation (7) is zero or negative resembles the standard properties of the steady state in the Solow (1956) model in the sense that, without technological progress, the long-run economic growth rate is zero. Denoting variables that are at the steady state with an asterisk, we would have the following:

$$\left[\frac{\dot{K}(t)}{K(t)} \right]^{*} = g_K^{*} = \text{ constant},$$

$$\left[\frac{\dot{P}(t)}{P(t)} \right]^{*} = g_P^{*} = \text{ constant},$$

$$g_K^* = s_m s \left[\left(\frac{K(t)}{L(t)+P(t)}\right)^*\right]^{-(1-\alpha)} - \delta$$

$$\Rightarrow \left[\frac{K(t)}{L(t)+P(t)}\right]^* = \left(\frac{s_m s}{g_K^* + \delta}\right)^{1/(1-\alpha)} = \text{constant},$$

$$g_P^* = (1-s_m)\, s \frac{1+[P(t)/L(t)]^*}{[P(t)/L(t)]^*}\left(\frac{s_m s}{g_K^*+\delta}\right)^{\alpha/(1-\alpha)} - \delta.$$

From the last equation it follows that $P(t)/L(t)$ has to be constant at the steady state. This implies that

$$g_P^* = \left[\frac{\dot{P}(t)}{P(t)}\right]^* = \frac{\dot{L}(t)}{L(t)} = n.$$

Due to the fact that

$$\frac{K(t)}{L(t)+P(t)} = \frac{\frac{K(t)}{L(t)}}{1+\frac{P(t)}{L(t)}}$$

is constant at the steady state, it follows that $K(t)/L(t)$ also has to be constant at the steady state. This implies that

$$g_K^* = \left[\frac{\dot{K}(t)}{K(t)}\right]^* = \frac{\dot{L}(t)}{L(t)} = n$$

and, in turn, that the growth rate of per capita GDP is zero.

A.5 Capital and labor income shares

Aggregate labor income in the economy is given by

$$w(t)L(t) = (1-\alpha)\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha} L(t), \tag{18}$$

while aggregate capital income is given by

$$r(t)K(t) + [w(t)-\delta]P(t) = \\ \left\{\alpha\left[\frac{L(t)+P(t)}{K(t)}\right]^{1-\alpha} + (1-\alpha)\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha} - 2\delta\right\}[K(t)+P(t)]. \tag{19}$$

This implies that the labor income share pins down to

$$\frac{w(t)L(t)}{Y(t)} = \frac{(1-\alpha)L(t)}{[L(t)+P(t)]^{1-\alpha}}, \tag{20}$$

while the capital income share is given by

$$\frac{r(t)K(t)+[w(t)-\delta]P(t)}{Y(t)}=$$
$$\frac{\left\{\alpha\left[\frac{L(t)+P(t)}{K(t)}\right]^{1-\alpha}+(1-\alpha)\left[\frac{K(t)}{L(t)+P(t)}\right]^{\alpha}-2\delta\right\}[K(t)+P(t)]}{[L(t)+P(t)]^{1-\alpha}K(t)^{\alpha}}. \tag{21}$$

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