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by

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Redistributive effects of the US pension system among individuals with different life expectancy*

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Abstract

We investigate the differential impact that pension systems have on the labor supply and the accumulation of physical and human capital for individuals that differ by their learning ability and levels of life expectancy. Our analysis is calibrated to the US economy using a general equilibrium model populated by overlapping generations, in which all population groups interact through the pension system, the labor market, and the capital market. Within our framework we analyze the redistributive and macroeconomic effects of a progressive versus a flat replacement rate of the pension system.

Keywords: Human capital, Longevity, Inequality, Life cycle, Social Security
JEL codes: E24, J10, J18, H55

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1 Motivation

Heterogeneity in life expectancy by socioeconomic status is increasing within countries. For instance, in the US the life expectancy gap between individuals with less than 12 years of schooling and with more than 16 years of schooling increased from less than 3 years in 1990 to 10 years in 2008 (Olshansky et al., 2012). This demographic trend leads to new concerns for the sustainability of the welfare state and urges to revisit whether existing policies reduce old-age inequality. At least, two questions need to be answered: Is the welfare state diminishing or increasing income inequality at old-age between individuals with different life expectancy? What are the economic implications of increasing differences in mortality across socioeconomic groups?

The principle of reducing income inequality at old-age is present in many pension systems. Some pension systems distribute resources at the tails of the income distribution, through the introduction of minimum and maximum benefit caps, while others distribute from high income earners to low income earners, through a progressive pension replacement rate. In the US, for example, the principle of giving an adequate safety net for individuals in the lower part of the income distribution is one of the cornerstones of the Social Security.[1] This goal is implemented through a progressive replacement rate (Golosov et al., 2013). However, the recent increasing gap in life expectancy by socioeconomic status can undermine the progressivity of the system. Individuals with a higher income, although they receive proportionally lower benefits, collect their pension benefits over a longer period of time given their higher life expectancy. In this regard, Liebman (2002), Goda et al. (2011), and recently National Academies of Sciences, Engineering, and Medicine (2015) show, using microsimulation models, how the progressivity of the US Social Security system is undone, and sometimes reversed, when the differential mortality either by educational attainment or by lifetime earning quintiles is taken into account. For instance, the National Academies of Sciences, Engineering, and Medicine (2015) reports that the present value of projected net benefits from age 50 onwards would be almost equal for males born in 1960 in the bottom quintile of lifetime earnings and for those in the top quintile. Hence, the progressivity of the US Social Security at age 50 for the cohort born in 1960 has been lost.

At the macro-level, the increasing inequality in longevity by socioeconomic status also raises concerns for the sustainability of the pension system and

[1] In 1939, an amendment to the Social Security Act was introduced providing higher proportion of benefits for people with lower lifetime earnings compared to those with higher lifetime earnings (Biggs et al., 2009).

economic growth. In particular, if, in the next decades, the present value of net benefits becomes higher for long living individuals than for those with shorter life spans, the Social Security spending will increase even faster than otherwise. Since individuals with higher incomes become progressively more costly to the pension system. Moreover, if a greater proportion of the national income will be devoted to support the elderly, labor supply will fall —as individuals will be more heavily taxed— and hence lifetime disposable income will decrease, leading to a reduction in savings and a slow down in economic growth.

In this paper we investigate the impact of two different pension systems on income redistribution at the micro and macro level in an economy where individuals differ with respect to life expectancy and consequently also their economic status.[2] We focus our analysis on the US economy given that its population witnesses an increasing life expectancy gap between different socio-economic groups. The report of the National Academies of Sciences, Engineering, and Medicine (2015) only focuses on the benefits of the US pension system. To also capture the evolution of contributions to pension systems over time, we base our results on a computable overlapping generation model with realistic demography, in which individuals optimally choose their labor supply, financial wealth, and human capital. As it is known from the literature, investment in human capital is not only due to differences in life expectancy, but also depends on differences in learning ability. We therefore combine the heterogeneity in life expectancy with heterogeneity in learning ability. This is done by assuming that individuals may belong to different frailty groups (Vaupel et. al., 1979) on the one hand, and that there exists a negative correlation between ability and frailty on the other hand.[3] Furthermore, we assume a retirement age equal for all individuals regardless of their life expectancy. This assumption resembles the current practice in many pension systems where the pensionable retirement age is independent of the expected life expectancy (OECD, 2011). Moreover, we assume that all individuals retire at the same age so as to focus on how different pension systems redistribute resources among individuals with different retirement length (as a result of different life expectancies). Labor supply at the intensive margin and human capital investment are endogenously modeled because social institutions intervene with heterogeneous life spans by affecting the incentives of individuals to invest in education and labor market participation through several mechanisms. First, according to Ben-Porath (1967) an increase in life expectancy goes together with an increase in the returns to ed-

[2]While heterogeneity in life expectancy and learning ability is assumed to be exogenously fixed, the resulting economic status is endogenously determined within a lifecycle model.

[3]The heterogeneity with respect to frailty and ability allows us to endogenously generate empirically observed variations of human capital investment.

ucation and hence an increase in educational investment. Second, an increase in life expectancy implies a longer period of consumption and also a greater chance to receive a future income, or life-time human wealth, which affects labor supply and the educational investment (see e.g. d'Albis et al. (2012); Cervellati and Sunde (2013); Sanchez-Romero et. al. (2016)). Third, following Ludwig and Reiter (2010) and Sanchez-Romero et. al. (2013) pension systems will interact with the increasing longevity inequality by creating different incentives for labor supply and educational investment by socioeconomic status through the effective social contribution rate.

The redistributive effects of the pension system among individuals with different life expectancy was investigated by Pestieau and Ponthiere (2012), who demonstrated, using an OLG model with two periods and two types of agents (long and short lived), that a social planner who maximizes the sum of individual utilities may induce a redistribution from the short to the long lived individuals. More recently, the role of heterogeneity in longevity and its implications for earnings-related pensions has been discussed by Ayuso et al. (2016), finding that the pension benefits of long lived and well-educated individuals are being subsidized by short lived and less educated individuals. To contribute to the literature, we compare the main retirement benefit program of the US pension system (OASDI), which is based on a progressive replacement rate (Golosov et al., 2013), to a pension system with a non progressive replacement rate, such as those in many European countries.

Our results indicate that in a pension system with flat replacement rate long lived and well educated individuals are partly subsidized by short lived and less educated individuals. In principle, with the US pension system this effect should be reversed. However, we also find that the widening life expectancy gap gives rise to this subsidy in the US. At the macro level, by comparing the income per adult in a non progressive pension system to that in a progressive pension system, we find that the income per adult is enhanced with a non progressive pension system relative to that in a progressive pension system. Nevertheless, the higher income per adult goes at the expense of increasing inequality. We show that the larger inequality caused by a non progressive pension system, such as those in many European countries, is due to the positive effect that this pension system has on the marginal benefit of education relative to that in a progressive pension system.

The paper is organized as follows: Section 2 introduces the model setup. Section 3 lists the equilibrium conditions. In Section 4, we introduce the parameterization of our model. In the following sections we solve the model numerically and consider the role of different pension systems for redistributive (section 5) and macroeconomic effects (section 6) in detail. Section 7 concludes the paper.

2 Model

As the framework of our economic analysis we choose a computable overlapping generation model populated by individuals that differ with respect to their life expectancy and learning ability. Individuals choose the optimal labour supply, human capital formation, and physical capital accumulation over their life cycle depending on their life expectancy and learning ability. We alternatively assume that individuals are confronted with a pension system that implements a progressive replacement rate versus a flat replacement rate.

2.1 Demographics

Time is discrete. Individuals enter the economy at the age of 15, face mortality risk, and may live up to a maximum of 120 years (denoted by Ω). We assume agents are heterogenous by their frailty level (μ) and by their learning ability level (θ), which implies heterogeneity for life cycle decisions such as human capital investment, labor supply and asset accumulation. We assume three frailty groups, $\mu \in M = \{1, 2, 3\}$. The first group ($\mu = 1$) is assumed to be the most frail and has the shortest longevity. Individuals that belong to the second group ($\mu = 2$) have an average frailty and hence their life expectancy is close to the average of the whole population. Individuals belonging to the third group ($\mu = 3$) are the less frail and have the longest life expectancy. Let Θ be the set of learning ability levels. We denote by $\mathcal{R}$ the region of all possible combinations of $\mu \in M$ and $\theta \in \Theta$. Let the probability of surviving to age j in year t of an individual of type μ be

$$s_{t,j}(\mu) = \prod_{u=15}^{j-1} \pi_{t-j+u,u}(\mu) \text{ for } j > 15, \text{ with } s_{\cdot,15}(\mu) = 1 \text{ and } s_{\cdot,\Omega}(\mu) = 0. \tag{1}$$

$\pi_{t,u}(\mu)$ is the conditional probability of surviving to age u for an individual in year t that belongs to the frailty group μ.

Let $N_{t,j}(\theta, \mu)$ be the number of people of type (θ, μ) who are j years old at time t. We assume for simplicity that our population is closed to migration and that individuals belonging to each demographic group (θ, μ) stay in that group until death. The dynamics of the population group (θ, μ) is described by

$$N_{t,15}(\theta, \mu) = G(\theta, \mu)\mathcal{N}_t, \tag{2}$$

$$N_{t+1,j+1}(\theta, \mu) = N_{t,j}(\theta, \mu)\pi_{t,j}(\mu) \text{ for } j \geq 15. \tag{3}$$

Eq. (2) is the population size of type (θ, μ) at age 15 in year t. $G(\theta, \mu)$ is the joint distribution of individuals of type (θ, μ) at age 15 and $\mathcal{N}_t$ is the total population at age 15 in year t. We assume $\mathcal{N}_t$ changes annually at the exogenously given rate n_t. Eq. (3) accounts for the total number of survivors of type (θ, μ) to time $t+1$ for the cohort entering the economy in year $t-j+15$. The total population size in year t equals the sum across age of the number of people who have survived from age 15 to year t by frailty level; i.e., $N_t = \sum_{j=15}^{\Omega} \mathcal{N}_{t+15-j} \int_M s_{t,j}(\mu) dg(\mu)$, where $g(\mu) = \int_\Theta dG(\theta, \mu)$ is the probability of being of type μ and $g(\theta) = \int_M dG(\theta, \mu)$ is the probability of being of type θ.

2.2 Firms

Firms operate in a perfectly competitive environment and produce one homogeneous good, which can be consumed or stored by individuals, according to a Cobb-Douglas production function:

$$Y_t = K_t^\alpha (A_t H_t)^{1-\alpha}, \tag{4}$$

where α is the capital share, Y_t is the output in period t, K_t is the stock of physical capital in period t, A_t is the labor-augmenting technological progress, and H_t is the aggregate stock of employed human capital in period t. For simplicity, we assume A_t increases annually at a constant rate g_A.

The capital stock evolves according to the law of motion $K_{t+1} = K_t(1-\delta) + I_t$, where δ is the depreciation rate of capital and I_t is aggregate gross investment. Production factors are paid their marginal products:

$$R_t^H = (1-\alpha)(Y_t/H_t), \tag{5}$$

$$r_t = \alpha (Y_t/K_t) - \delta, \tag{6}$$

where R_t^H is the rental rate on human capital at time t and r_t is the net return on physical capital at time t.

2.3 Agent's problem

Agents at time t differ in six dimensions. Three dimensions that are exogenously fixed —age (j), the learning ability (θ), and the frailty group (μ)— and three characteristics that result from the optimization process —asset holdings (a), the stock of human capital (h), and the average past pension earnings (p). Let us denote the set of the state variables at age j at time t for an agent of type (θ, μ) by $X_{t,j}(\theta, \mu) = \{a_{t,j}, h_{t,j}, p_{t,j}\}$. The expected utility (V) of a household head of type (θ, μ) and age j at time t takes the following functional form:

$$V_{t,j}\left(X_{t,j}(\theta, \mu)\right) = U\left(c_{t,j}, z_{t,j}\right) + \beta\pi_{t+1,j+1}(\mu)V_{t+1,j+1}\left(X_{t+1,j+1}(\theta, \mu)\right) \tag{7}$$

where β is the subjective discount factor, U is the period utility function (with $U_c \geq 0, U_z \geq 0, U_{cz} \leq 0, U_{cc} \leq 0,$ and $U_{zz} \leq 0$), $c_{t,j}$ is the consumption and $z_{t,j}$ is the leisure time of an agent at age j in year t.

We assume agents start making decisions at the age of 15, which corresponds to the age after nine years of compulsory education. Each period, agents are endowed with one unit of time. They optimally choose their consumption path, leisure time, hours of work, and the fraction of time invested in human capital formation. Agents start with zero assets, zero pension earnings, and an initial human capital h_0 that is similar for all individuals regardless their life expectancy and ability due to the compulsory education (i.e. $a_{t,15} = p_{t,15} = 0$ and $h_{t,15} = h_0$ for all t). Following Yaari (1965) we assume a perfect annuity market in which agents can purchase life-insured loans when they are in debt, which allow them to borrow against future labor income, and also can purchase annuities in case of having positive financial wealth. Thus, assets held (a) evolves over the life cycle according to

$$a_{t+1,j+1} = \begin{cases} R_{t,j}(\mu)a_{t,j} + (1-\tau_t)y_{t,j} - c_{t,j} & \text{for } j \leq J_R, \\ R_{t,j}(\mu)a_{t,j} + b_{t,j} - c_{t,j} & \text{for } j > J_R, \end{cases} \tag{8}$$

where $R_{t,j}(\mu) = (1 + r_t)/\pi_{t,j}(\mu)$ is the capitalized rate of return, contingent on survival, of one unit of capital invested at age j in year t, τ_t is the social security contribution rate in year t, $y_{t,j} = R_t^H h_{t,j}\ell_{t,j}$ is the (gross) labor income at age j in year t, which is a function of the rental rate of human capital in year t (R_t^H), the stock of human capital at age j in year t $(h_{t,j})$, and the fraction of time devoted to work $(\ell_{t,j})$; J_R is the retirement age, and $b_{t,j}$ is the pension benefits received at age j in year t by an individual retired at age J_R, which can be decomposed in the following two components:

$$b_{t,j} = \psi(p_{t,j})p_{t,j}, \tag{9}$$

where $\psi(p_{t,j})$ is the replacement rate associated to $p_{t,j}$ pension earnings.

The introduction of pension earnings as a state variable in Eqs. (7)-(8) implies that agents understand the rules on how pension benefits are calculated (Ludwig and Reiter, 2010; Sanchez-Romero et. al., 2013). Hence, agents internalize that higher labor earnings affects positively on their future pension benefits according to

$$p_{t+1,j+1} = \begin{cases} I_t p_{t,j} + \varrho_j y_{t,j} & \text{for } 15 \leq j \leq J_R, \\ p_{t,j} & \text{for } j > J_R, \end{cases} \quad \text{with } p_{t,15} = 0 \ \forall t, \tag{10}$$

where I_t is a weight factor on past pension earnings and ϱ_j is the weight of labor income at age j on pension earnings.

Agents may devote time to education, denoted by e, to increase their future human capital and hence labor income. We assume human capital accumulates according to a standard Ben-Porath (1967) technology

$$h_{t+1,j+1} = \begin{cases} h_{t,j}(1-\delta_h) + q\left(h_{t,j}, e_{t,j}; \theta\right) & \text{for } 15 \leq j \leq J_R, \\ h_{t,j}(1-\delta_h) & \text{for } j > J_R. \end{cases} \tag{11}$$

That is, human capital increases due to investments in human capital $q(h, e; \theta)$ (with $q_h \geq 0, q_e \geq 0, q_{he} \leq 0, q_{hh} \leq 0,$ and $q_{ee} \leq 0$), and decreases due to the depreciation of human capital at an annual rate δ_h. We use a standard Ben-Porath human capital production function

$$q\left(h_{t,j}, e_{t,j}; \theta\right) = \varphi(\theta)\left(h_{t,j} e_{t,j}\right)^{\gamma} \text{ with } \varphi > 0, \gamma \in (0,1), \tag{12}$$

where $\varphi(\theta)$ is the learning ability for the individual of type θ and γ is the returns to scale in human capital investment.

2.3.1 Agent's decision problem

Agents optimally allocate the resources by maximizing (7) with respect to consumption, leisure, and human capital investment subject to (8)-(11) and the time constraints: $z_{t,j} \geq 0, \ell_{t,j} \geq 0, e_{t,j} \geq 0$, and $z_{t,j} + \ell_{t,j} + e_{t,j} = 1$. All feasible solutions are derived in Appendix A. In this section, we focus on the pre-retirement period.[4]

[4]During retirement our agents only decide about their consumption path. Leisure is equal to one, and therefore both the hours worked and the human capital investment are equal to zero.

The following first-order conditions govern the model:

$$U_{c_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}}, \tag{13}$$

(the marginal utility of consumption is equal to the marginal cost of saving)

$$U_{z_{t,j}} \geq \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H h_{t,j}, \tag{14}$$

(the marginal utility of leisure is equal, or greater, than the marginal cost of working)

and

$$\frac{\partial V_{t+1,j+1}}{\partial h_{t+1,j+1}} \gamma \varphi(\theta) h_{t,j} (h_{t,j} e_{t,j})^{\gamma - 1} \geq \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H h_{t,j}, \tag{15}$$

(the marginal benefit of education is equal, or greater, than the marginal cost of working)

Note that a strict inequality in Eq. (15) implies that agents specialize in schooling and devote all their time between schooling and leisure. Hence, we can distinguish up to three periods, which are endogenously determined, of human capital investment from the first-order conditions: i) a period of specialization in schooling —when the marginal benefit of schooling is greater than the marginal cost of working—, ii) a period of work and education —when Eq. (15) is satisfied with equality— in which the time devoted to human capital investment monotonically decreases until retirement, and iii) a retirement period in which individuals do not invest in human capital. The existence of these endogenously determined periods were proved by Blinder and Weiss (1976). The following envelope conditions also hold:

$$\frac{\partial V_{t,j}}{\partial a_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} R_t, \tag{16}$$

(intertemporal arbitrage in returns on physical capital)

$$\frac{\partial V_{t,j}}{\partial h_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \left(\frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H \ell_{t,j} + \frac{\partial V_{t+1,j+1}}{\partial h_{t+1,j+1}} \frac{\partial h_{t+1,j+1}}{\partial h_{h,j}} \right), \tag{17}$$

(the marginal value of human capital is the return to current and future earnings)

$$\frac{\partial V_{t,j}}{\partial p_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} I_t, \tag{18}$$

(the marginal value of pension earnings is the return to future pension earnings)

where $\tilde{\tau}_{t,j}$ is the effective social security tax rate, which is given by

$$\tilde{\tau}_{t,j} = \tau_t - \rho_j \left. \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} \middle/ \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} \right. . \tag{19}$$

From Eq. (19) we have that the effective social contribution rate is the difference between the social security contribution rate and a fraction ρ_j of the marginal rate of substitution between public pension stock and asset holdings. Thus, agents perceive only part of the social security contribution rate as a tax and it can even be seen as a subsidy when $\tau_t \leq \rho_j \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} \Big/ \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}}$.

From (13) and (16) the optimal consumption path for an individual of type μ is given by

$$\frac{U_{c_{t,j}}}{U_{c_{t+1,j+1}}} = \beta(1 + r_{t+1}), \tag{20}$$

where Eq. (20) is the well-known Euler condition. The left-hand side of Eq. (20) is the marginal rate of substitution between present and future consumption, while the term on the right-hand side implies that consumption increases when agents discount future consumption less than the market, i.e. $\beta(1 + r_{t+1}) > 1$, and decreases when $\beta(1 + r_{t+1}) < 1$. Thus, for the same ability level, the existence of three frailty groups in the population implies three different consumption, labor supply, and educational investment trajectories.

The optimal labor supply decision is characterized by the marginal rate of substitution between consumption and leisure:

$$\text{MRS}_{c,z} \equiv \frac{U_{z_{t,j}}}{U_{c_{t,j}}} \geq (1 - \tilde{\tau}_{t,j}) R_t^H h_{t,j} \qquad \text{for } j \in \{15, \ldots, J_R\}. \tag{21}$$

Eq. (21) implies that leisure is increasing the larger is the net (of implicit labor tax) wage rate per hour worked.

From (12) and (15)-(17), the optimal educational investment satisfies

$$e_{t,j} = \frac{1}{h_{t,j}} \left(\frac{\gamma\varphi(\theta)}{1-\delta} \sum_{x=t+1}^{t+J_R-j} \left[\prod_{i=t+1}^{x} \frac{1-\delta}{1+r_i} \right] \frac{s_{x,x-t+j}(\mu)}{s_{t,j}(\mu)} \frac{1 - \tilde{\tau}_{x,x-t+j}}{1 - \tilde{\tau}_{t,j}} \frac{R_x^H}{R_t^H} (1 - z_{x,x-t+j}) \right)^{\frac{1}{1-\gamma}}. \tag{22}$$

According to (22) the optimal fraction of time devoted to education is increasing the higher is the learning ability of the individual, the smaller the future leisure time, the higher the survival probability, the higher the retirement age (J_R), the lower the human capital depreciation rate, and the lower the future returns on physical capital. Moreover, it is worth stressing that the pension system may also influence the optimal investment in education through the effective social security tax rate. Thus, in our framework, there exists a positive relationship between the educational investment and the pension system when future effective social contribution rates are lower than the present effective social contributions rates, i.e. $\tilde{\tau}_{t-j+x,x} < \tilde{\tau}_{t,j}$ for $j \leq x \leq J_R$. See Appendix B.1 for a detailed explanation.

2.4 Government

The government runs a balanced pay-as-you-go (PAYG) pension system. The social contribution rate is the same for all type of agents regardless their age, learning ability, or life expectancy. The government sets a mandatory retirement age J_R, which it is assumed to be the same in all periods. To guarantee a zero deficit in the PAYG pension system, the government is assumed to modify the social security contribution rate τ_t each year in order to finance all the pension benefits claimed by retirees. Thus, the budget of the pension system is

$$\tau_t R_t^H H_t = \sum_{j=J_R+1}^{\Omega} \mathcal{N}_{t+15-j} \int\int_{\mathcal{R}} b_{t,j}(\theta,\mu) s_{t,j}(\mu) dG(\theta,\mu). \tag{23}$$

(Total contributions paid equal total pension claimed)

3 Equilibrium conditions under perfect foresight

By assuming differences in learning ability (μ) and life expectancy (θ), agents will differ because of different prices over their life course. Let $\mathcal{R}$ be the region of all possible combinations of μ and θ. Let $G(\theta,\mu)$ be the joint distribution of individuals of type (θ,μ) at age 15. Let P_s be the vector of rental prices of physical capital and human capital, and social security contribution rates faced by an individual born in year s over the lifecycle.

Given initial economic values $\{\alpha, g_A, \delta, \varphi(\theta), \gamma, \mu\}$ and demographic data $\{\mathcal{N}_t, \pi_{t,j}(\mu)\}$ over time, we can define the recursive competitive equilibrium as the sequence of a set of households policy functions $\{X_{t,j}(\theta,\mu), c_{t,j}, z_{t,j}, e_{t,j}\}$, government policy functions $\{\tau_t, \psi(p)\}$, and factor prices $\{R_t^H, r_t\}$, for $j \in \{15,\ldots,\Omega\}$ and $t > 0$, such that

1. Agents policy functions satisfy Eqs. (13)-(18)

2. Factor prices equal their marginal productivities so that Eqs. (5) and (6) hold.

3. The government's budget constraints in Eq. (23) is satisfied.

4. The aggregate stock of physical capital and the aggregate stock of em-

ployed human capital are given by:

$$K_t = \sum_{j=15}^{\Omega} \mathcal{N}_{t+15-j} \int \int_{\mathcal{R}} a_{t,j}(\theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta, \mu), \tag{24}$$

$$H_t = \sum_{j=15}^{\Omega} \mathcal{N}_{t+15-j} \int \int_{\mathcal{R}} h_{t,j}(\theta, \mu, P_{t-j}) \ell_{t,j}(\theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta, \mu). \tag{25}$$

5. The commodity market clears:

$$Y_t = C_t + S_t, \tag{26}$$

where $C_t = \sum_{j=15}^{\Omega} \mathcal{N}_{t+15-j} \int \int_{\mathcal{R}} c_{t,j}(\theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta, \mu)$ is the total consumption in year t and S_t is the gross saving in year t.

4 Parametrization

To keep the model as tractable as possible, we assume that individuals upon entering the economy are randomly assigned to a learning ability and frailty group according to a distribution $G(\theta, \mu)$ over these two characteristics. The distribution $G(\theta, \mu)$ comprises the combination of three possible learning abilities and three frailty groups. This gives a total of nine different population groups within each cohort. The reason for having three cases in each characteristic rather than two is to be able to analyze the average individual at the same time as having greater heterogeneity within a computationally tractable model.

We run two alternative scenarios. A baseline that replicates the current parametric components of the US pension system and second, a counterfactual experiment with a pension system that applies a flat replacement rate to all retirees, whose value corresponds to the average replacement rate of the total retired population in the baseline scenario.

Next, we explain the main assumptions introduced in the demographic and the economic setup to disentangle the different effects.

4.1 Demographics

We replicate the overall demographic features of the US population from year 1850 to 2010 –i.e., single age-specific fertility rates and single age-specific mortality rates– using the generalized-inverse population projection method (Lee, 1985; Oeppen, 1993). The demographic information for the period 1850-2010 is taken from the US Bureau of the Census (1949), the Human Mortality Database (2015), and the Human Fertility Database (2015). Future fertility rates for the US population are based on UN, Population Division (2013). On top of the derived mortality rates from the population reconstruction, we introduce lifespan heterogeneity and project future mortality rates using the Lee-Carter model (Lee and Carter, 1992).[5] Before year 1800 we assume a stable population (i.e. constant population growth). From 1800 to 1850, we have a transition period from a stable population in 1800 to the population observed in 1850. At the individual level, we assume recently born individuals differ by their learning ability and life expectancy. We consider three learning abilities $\theta = \{$*low=1, average=2, high=3*$\}$ and three alternative life expectancies $\mu = \{$*high=1, average=2, low=3*$\}$, with equal marginal probabilities. In order to capture the positive correlation between the life expectancy and the learning ability we give a population weight of 1/6 in the main diagonal, while we give a population weight of 1/12 for all groups outside of the main diagonal. Nonetheless, one should be aware of the fact that these weights will differ as individuals age. In particular, the proportion of less frail individuals will increase, the fewer surviving individuals are left in each birth cohort.

Figure 1 shows the evolution of the main aggregate demographic variables: life expectancy, total fertility rate, and population distribution for three selected years. Panel 1(a) shows the evolution of the life expectancies by frailty group, while Panel 1(b) shows the evolution of the life expectancy by learning ability. The life expectancy of the average individual, who is represented by the solid red curves in panels 1(a)-1(b), replicates the observed life expectancy of the population. Moreover, the positive correlation between life expectancy and ability can be seen in Panel 1(b). The difference in life expectancy by ability group is smaller than that by frailty due to the fact that each ability group is comprised of individuals with different mortality frailties. Panel 1(c)

[5] According to the Lee-Carter model, the temporal component of mortality gains at time t can be described, on average, according to the following time series

$$k_{t+1} = \hat{\mu} + \phi k_t. \tag{27}$$

In order to introduce the heterogeneity in our model, we assume individuals are subject to different drifts as follows

$$\hat{\mu} = \bar{\mu} + \mu \text{ with } \mu \sim \mathcal{U}(-\bar{\mu}, \bar{\mu}) \text{ and } 0 < \mu < \bar{\mu}. \tag{28}$$

Thus, $\bar{\mu}$ matches the observed drift in the population, whereas each population group would have a different μ value, and hence a different life expectancy. We apply this model from year 1900 onwards to the US population.

shows the evolution of the total fertility rate. Panel 1(d) shows the population distribution in years 2015, 2050, and 2100 that results from the fertility and mortality depicted in Panels 1(a)-1(c). Nevertheless, it is important to keep in mind that, although we replicate the population censuses, the population used in this paper only comprises individuals above age 15.

(a) Life expectancy by frailty (μ)

(b) Life expectancy by ability (θ)

(c) Total fertility rate

(d) Total population distribution

Figure 1: US demographics

4.2 The economy

In this subsection we briefly explain the US pension system (our baseline) and the counterfactual pension system implemented in our simulations. Second, we introduce the main model economy parameters.

The pension system. The US pension system started in year 1935. The main feature of the US pension system compared to many other pension systems of OECD countries is that its replacement rate is progressive. Thus, in order to fully understand its redistributive effects, our simulation results are based on the comparison of two alternative pension systems. First, one pension system whose pension formula resembles the largest 'retirement benefit program' of the US pension system (OASDI). For illustration, Figure 2 shows how the US pension replacement rate declines as the pension earnings —known as the Average Indexed Monthly Earnings (AIME)— increase relative to the average labor income. For a detailed exposition of the parametric components used in this article see Appendix B. Second, we assume a counterfactual pension system with a fix replacement rate. For comparative purposes between the baseline and the counterfactual, we set the fix replacement rate at 0.417, which is the replacement rate value for the average worker in the baseline simulation.

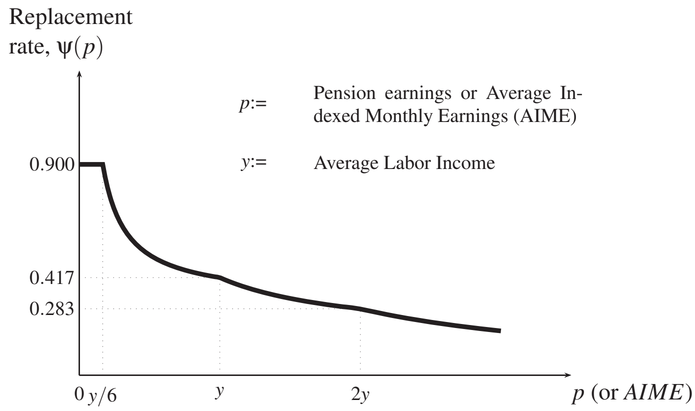


Figure 2: Old-Age Insurance replacement rate in the US

Note: AIME is calculated as 1/12 of the mean of the 35 highest labor incomes over the working life, measured in real terms.

Model economy parameters. Table 1 reports the model economy parameters, which are standard in the literature. We choose the learning ability values, φ, so as to have an entrance in the labor market in year 2015 at age

16 for low learning ability individuals, at age 18 for medium learning ability individuals, and at age 21 for those individuals with high learning ability.

Table 1: Model economy parameters

Parameter	Symbol	Value
Preferences		
Intertemp. elasticity of substitution	σ	0.500
Leisure weight	ϕ	1.000
Subj. discount factor	β	1.000
Returns to scale in education	γ	0.650
Learning ability	φ	{0.077;0.090;0.104}
Human capital depreciation	δ_h	0.008
Technology		
Capital share	α	0.33
Capital depreciation rate	δ	0.05
Labor-aug. tech. progress growth rate	g_A	0.02
Government		
Mandatory retirement age	J_R	65
Weight of current income on pensions	ϱ_j	$\begin{cases} 0 & j \leq J_R - 35 \\ 1/35 & J_R - 35 < j \leq J_R \end{cases}$

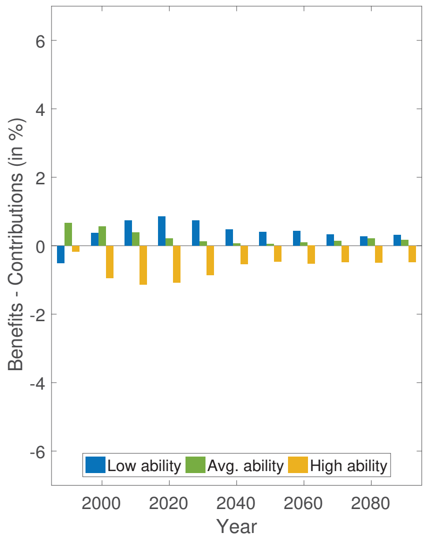
We run the model assuming a standard utility function

$$U(c_{t,j}, z_{t,j}) = \log c_{t,j} + \phi \frac{z_{t,j}^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \tag{29}$$

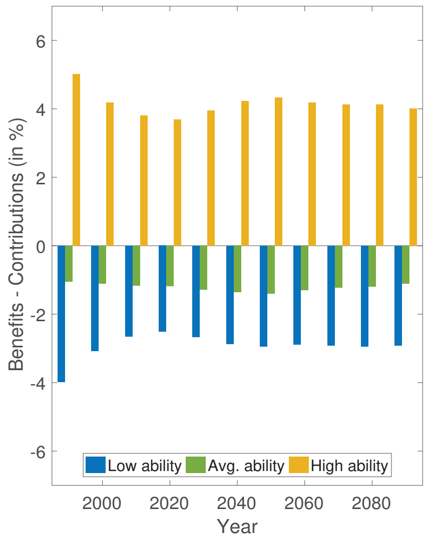
where ϕ represents the weight on the utility of leisure relative to the utility from consumption and σ is the intertemporal elasticity of substitution on leisure.

5 Redistributive effects: Progressive vs. flat replacement rate

This section focuses on the redistributive consequences of implementing a progressive replacement rate versus a flat replacement rate. An analysis of the behavioral effects of running these two replacement rates is presented in Appendix B.1. Using our simulations we are interested in knowing who are the net beneficiaries of each pension system. We start looking at how each simulation scenario redistributes resources among the three ability groups from a group perspective (cross-sectional) and from an individual perspective (cohort). The redistributive effects among the three life expectancy groups for all the following figures are shown in Appendix C.



(a) Baseline



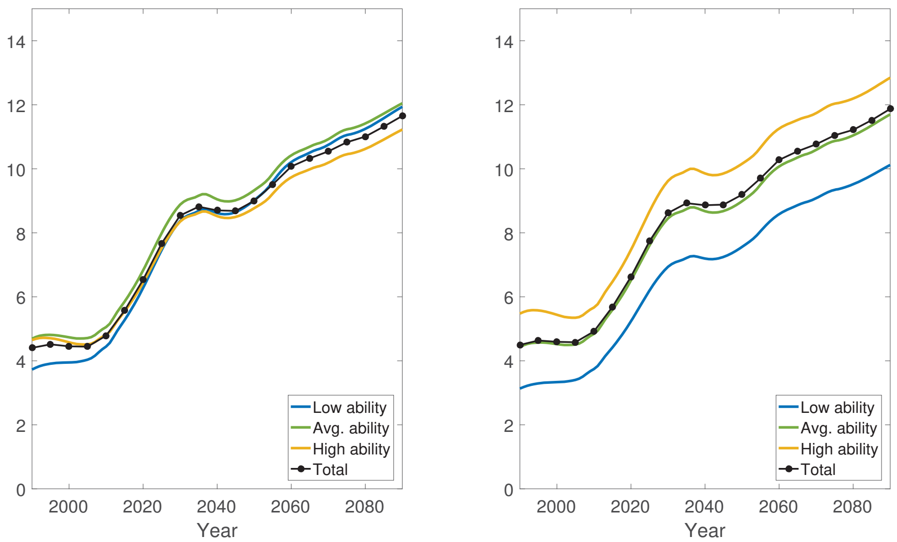
(b) Flat replacement

Figure 3: Benefits minus contributions by ability group (in percentage of the total pension budget): Calendar years 1990-2090

As already mentioned, the goal of the US pension system is to redistribute resources from high income individuals to low income individuals through the progressive replacement rate formula. To study the redistribution of resources, we first calculate the total benefits received minus contributions paid by each ability group θ in a given year t, which we denote by $SS_t(\theta)$, as

$$SS_t(\theta) = \sum_{\mu=1}^{3} \sum_{j=J_R+1}^{\Omega} b_{t,j}(\theta,\mu)N_{t,j}(\theta,\mu) - \tau_t \sum_{\mu=1}^{3} \sum_{j=15}^{J_R} y_{t,j}(\theta,\mu)N_{t,j}(\theta,\mu). \quad (30)$$

In our general equilibrium model with perfect foresighted individuals, Figure 3(a) shows how the US replacement rate formula (baseline) does almost not redistribute resources among our three ability groups. The difference between lifetime benefits and contributions is almost zero and equal across all ability levels. In contrast, Figure 3(b) shows how a constant replacement rate of 0.417 implies that the low and average ability groups transfer each year close to 4% of the total pension budget to the high ability group. As Figure 4(b) shows this is because the cost of pensions received by individuals with high ability, relative to their output produced, is the highest in the flat replacement rate scenario, while the lowest pension costs corresponds to the low ability group.



(a) Baseline simulation

(b) Flat replacement simulation

Figure 4: Total pension to output ratio from 1990 to 2090 (in %)

According to Figure 4 the total pension to output ratio will progressively increase from less than five percent at the beginning of the twenty first century to twelve percent in year 2090.[6] However, neither Figure 3 nor Figure 4 are informative at the individual level. To understand the redistributive goal of the pension system, we further calculate for each individual type the ratio between the present value of net benefits (benefits minus contributions) over the life cycle, also known as social security wealth, and the present value of

[6]It is important to bear in mind that we have assumed a fixed retirement age of 65 for a better understanding of the alternative effects. If the latest reforms of the OASDI pension system were implemented, in which the normal retirement age progressively increases for earlier cohorts, these figures would slightly change.

the stream of labor income as follows

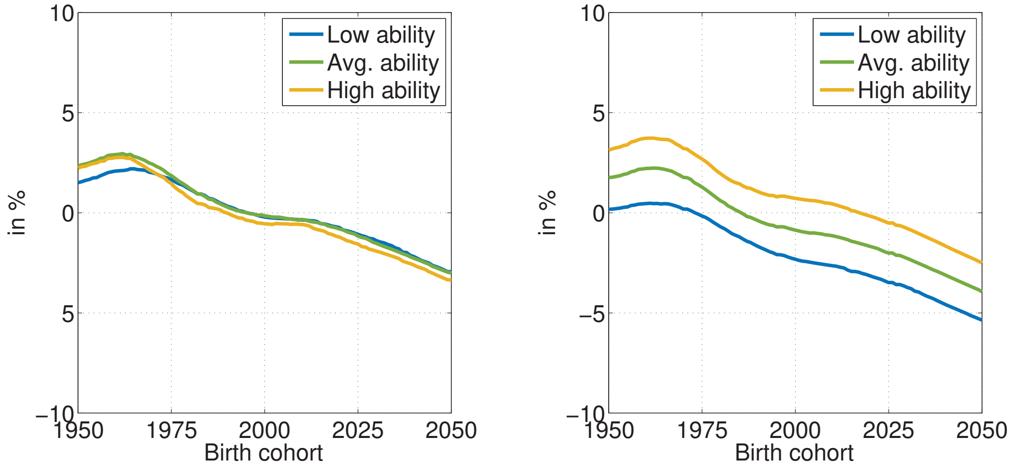
$$\int_M \left(\sum\nolimits_{j=15}^{\Omega} D_{c,j}(\mu)\text{Net Ben}_{c+j,j}(\theta,\mu)\Big/\sum\nolimits_{j=15}^{J_R} D_{c,j}(\mu)y_{c+j,j}(\theta,\mu)\right)\frac{dG(\theta,\mu)}{g(\theta)}, \tag{31}$$

where $D_{c,j}(\mu) = 1/\prod_{x=15}^{j} R_{c+x,x}(\mu)$ is the cumulated discount factor at age j of an agent born in year c. Thus, Eq. (31) informs us about the monetary gains or losses of participating in the social security system for each ability level θ.

Figure 5 shows the ratio between the social security wealth and the present value of labor income by ability level in the baseline simulation (Panel 5(a)) and for a flat replacement rate (Panel 5(b)). Results presented in Figure 5 lead to the following three conclusions: i) in the baseline simulation, the social security wealth as a fraction of the present value of labor income is very similar across the different ability groups; ii) with a flat replacement level, the social security wealth relative to the present value of labor income is smaller, the lower the ability level is. Given the different values, conclusion ii) implies that there exists a redistribution between the different ability groups. In particular, individuals with higher ability are subsidized by individuals with low ability. iii) for all ability levels and in both pension systems, younger birth cohorts will suffer a monetary loss from participating in the social security pension system. In the baseline simulation, cohorts born after 2000 can expect to contribute more than they receive regardless their learning ability level. In contrast, in a flat replacement rate, individuals with high learning ability will receive from the pension system more than they pay for almost forty additional years compared to the low learning ability group.

6 Macroeconomic effects: Progressive vs. flat replacement rate

According to Figure 4 the population aging process will raise the future cost of the OASDI pension system. Under a balanced budget, the higher social contribution rate may affect the prospects for economic growth through a reduction in savings, because of a decline in disposable income, and through a reduction in labor supply, because of a decline in the (net) wage rate. Moreover, as we showed in Figure 3, the increasing life expectancy gap between ability groups may lead the pension system to redistribute from individuals with low ability to individuals with high ability. As a consequence, the pension system may also increase income inequality across ability groups.



(a) Baseline

(b) Flat replacement

Figure 5: Social security wealth as a fraction of present value of labor income (in %): 1950-2050 birth cohorts

In this section we first study the evolution of output per adult by ability group and, second, we continue with the study of the income inequality across ability groups.

Output per adult. To understand the macroeconomic consequences of population aging under the two alternative pension systems, we decompose the output per adult (Y/N_{15+}) by ability level as follows[7]

$$(Y_t/N_{t,15+}) = \frac{R_t^H}{1-\alpha} \sum\nolimits_{\theta=1}^{3} h_t(\theta)\ell_t(\theta)\left(N_{t,15+}(\theta)/N_{t,15+}\right), \tag{33}$$

[7]From (5) and assuming perfect substitutability between skill groups, we have that the total labor income generated by workers is $(1-\alpha)Y_t = R_t^H H_t$, where H_t can be expressed as the sum of the income generated by each ability group

$$H_t = \sum\nolimits_{\theta=1}^{3} h_t(\theta)\ell_t(\theta)N_{t,15+}(\theta). \tag{32}$$

Substituting (32) in the total labor income and dividing both sides of the equation by $(1-\alpha)N_{t,15+}$ gives (33).

where

$$h_t(\theta) = \frac{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} h_{t,j}(\theta, \mu, P_{t-j}) \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}, \tag{34}$$

(Average human capital per worker of type θ)

$$\ell_t(\theta) = \frac{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} N_{t,j}(\theta, \mu)}. \tag{35}$$

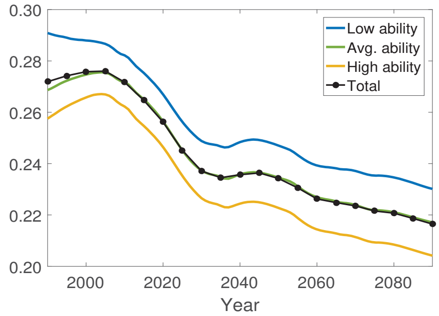
(Average time devoted to work per adult of type θ)

The first term on the right-hand side of Eq. (33) is the average output per unit of human capital, $(R_t^H/(1-\alpha))$, and the last term on the right-hand side is the population size of group θ in year t as a percentage of the total population $(N_{t,15+}(\theta)/N_{t,15+})$. Therefore, Eq. (33) allows us to analyze the impact of each pension system on output per adult by studying the effect of the pension system on the accumulation of human capital and on labor supply by ability level.

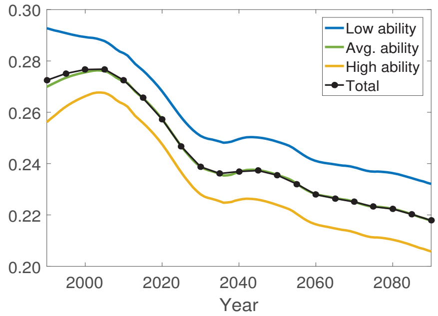
To see the macroeconomic consequences of running each pension system we use Eq. (33) stepwise. First, we show in Figure 6 the evolution of the components on the right-hand side of Eq. (33) and, second, we look at the output produced by each ability group in Table 2. Before we proceed to explain the results, it is important to bear in mind that in an economy with a changing population structure, the evolution of all aggregate variables is given by a change in the age distribution of each population group and by changes in the life cycle profiles, or behavioral reaction (Lee, 1980).

Given that individuals mandatorily retire at age 65, the top panels in Figure 6 show that the time spent working over the life cycle will decline on average from 28% to 22% due to the longer life expectancy. This is equivalent to a fall of 20% in the average working time by adults from year 1990 to 2090. Looking at the top panels in Figure 6 we can notice that the fall is very similar across ability groups. Comparing panel 6(a) to 6(b), we see that the pension system does not significantly affect the intensive labor supply across the different ability groups, since we obtain the same results independent of the replacement rate considered. However, according to panels 6(c) and 6(d), the decline in the intensive labor supply over the life cycle will be offset by an increase of around 50% in the average human capital employed from 1990 to 2090.[8] With a flat replacement rate formula the effective social contribution

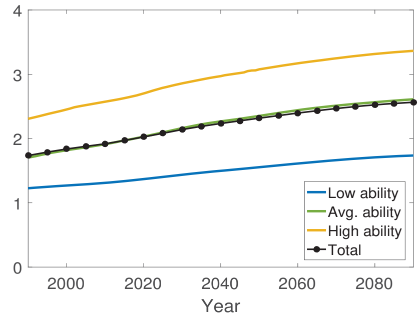
[8] Comparing the average human capital employed, $h_t(\theta)$, in year 1990 to that in 2090, which are shown in 6(c) and 6(d), we have that $h_t(\theta)$ increases in the flat replacement rate scenario by 55% and in the baseline scenario by 49%.



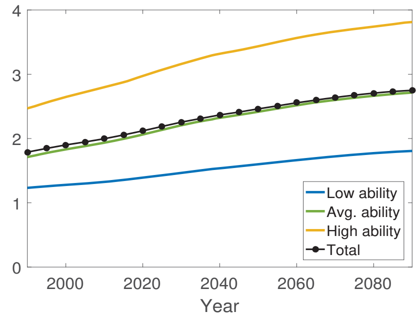
(a) Avg. working time by adults $\ell_t(\theta)$, baseline



(b) Avg. working time by adults $\ell_t(\theta)$, flat replacement



(c) Avg. human capital employed $h_t(\theta)$, baseline



(d) Avg. human capital employed $h_t(\theta)$, flat replacement

Figure 6: Decomposition of the output per adult by ability level: Period 1990-2090.

rate raises the marginal benefit of education to a higher extent than in case of the progressive replacement rate and hence, the educational investment, see Eq. (22).

Multiplying the increase in human capital by the evolution of the average hours worked and the rental rate of human capital, we show in Table 2 the increase in output per adult by ability from 1990 to 2090. Overall, the output per adult increase from 1990 to 2090 slightly less than 30% (after de-trended by an annual productivity growth of 2 percent) in the baseline scenario and slightly more than 30% with a flat replacement rate.

Income inequality. At the aggregate level, the value of production equals the income paid to the production factors. In contrast, in an economy with different groups, the output produced by each group does not necessarily coin-

Table 2: Growth of output per adult by ability group (1990=1.00)

	Baseline				Flat replacement			
Year	2015	2040	2065	2090	2015	2040	2065	2090
$Y_t(\theta)/N_{t,15+}(\theta)$								
Low	1.07	1.09	1.16	1.21	1.07	1.11	1.19	1.25
Avg.	1.17	1.21	1.29	1.33	1.17	1.23	1.32	1.37
High	1.17	1.17	1.22	1.24	1.20	1.23	1.30	1.33

Source: Author's calculations. Note: Values of $Y_t(\theta)/N_{t,15+}(\theta)$ are productivity de-trended.

cide with the income received by the same group. Notice that in a neoclassical production function, given that the capital per worker does not differ across ability groups, the difference in output per worker is driven by the difference in human capital. However, the income received by each ability group depends both on the human capital stock and on savings. Thus, for those individuals whose savings-to-output ratio exceeds those of the average, the income received exceeds the output generated and vice versa. Moreover, as we have seen in Section 5, the Social Security can redistribute resources across the different ability groups, which may enhance or reduce the difference between output and income. Eq. (36) shows for any ability group θ how the savings-to-output ratio $(K_t(\theta)/Y_t(\theta))$ and the Social Security $(SS_t(\theta))$ influence the relationship between income per adult —measured on the expenditure side by the sum of consumption $C_t(\theta)$ and investment $I_t(\theta)$— and output per adult (see the proof in Appendix D)

$$\underbrace{\frac{C_t(\theta)+I_t(\theta)}{N_{t,15+}(\theta)}}_{\textbf{Income per adult}} = \underbrace{\frac{Y_t(\theta)}{N_{t,15+}(\theta)}}_{\textbf{Output per adult}} \left(1+\underbrace{\Phi_{1,t}(\theta)}_{\textbf{Factor I}}+\underbrace{\Phi_{2,t}(\theta)}_{\textbf{Factor II}}\right) \tag{36}$$

where

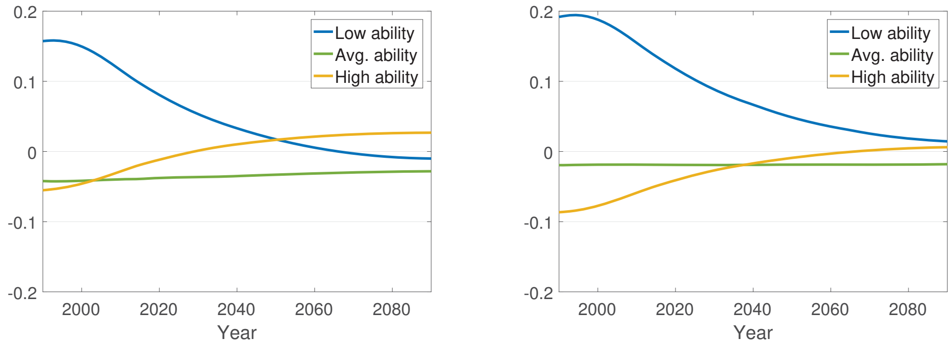
$$\textbf{(Factor I)} \qquad \Phi_{1,t}(\theta) = \alpha\left[(K_t(\theta)/Y_t(\theta))/(K_t/Y_t)-1\right], \tag{37}$$

$$\textbf{(Factor II)} \qquad \Phi_{2,t}(\theta) = SS_t(\theta)/Y_t(\theta). \tag{38}$$

Although our simulations have shown in Table 2 that the growth of output per adult differs between the three ability groups by less than 12% (after detrending by productivity) from 1990 to 2090, Eq. (36) implies that income inequality can still increase or decrease in the decades to come depending on the evolution of factors I and II. Factor I accounts for the contribution of the difference between the saving-to-output ratio of each ability group $(K_t(\theta)/Y_t(\theta))$ and that of the average individual (K_t/Y_t) to the gap between income and output. While factor II takes into account the net contribution of the social security $(SS_t(\theta)/Y_t(\theta))$ to the gap between income and output. We analyze the evolution of each factor in figures 7 and 8.

Figure 7 shows the evolution of factor I for the three ability groups from 1990 to 2090. Assuming that factor II is equal to zero, a positive (resp. negative) value of factor I implies that the income earned exceeds (resp. is lower than) the output produced. Thus, the blue solid line shows how individuals with low ability earn around 15% (baseline) and 20% (flat replacement) more than the output they produced in 1990. In contrast, individuals belonging to the high ability group earned around 5% (baseline) and 9% less than their output generated in 1990. By year 2090, Figure 7 shows that the difference between these two groups vanishes with a flat replacement rate and it is reversed in the baseline simulation. There are two facts that explain the evolution of these two trends. First, over the life cycle the development of the savings-to-output ratio $(K_t(\theta)/Y_t(\theta))$ is different across the three ability groups. Before age 40, individuals with low ability have more savings than those with higher ability, since the former have longer working histories and they do not need to finance their schooling period. Moreover, individuals with low ability have lower human capital accumulated since they have less years of schooling. Consequently, the savings-to-output ratio of low ability individuals is higher around age 40 than that of individuals with higher ability. Before retirement, in contrast, individuals with higher ability have higher savings. While the rate of increase in human capital before retirement is small in all groups. The second fact is that over time the mean age of the adult population changes due to the aging process. In particular, the mean age of adults was in the 1990s around 43 years. Thus, individuals with high ability at this age have a lower savings-to-output ratio than those with low ability. In 2090, the mean age of adults ranges between 53 (for the low ability group) and 56 years (for the high ability group). Thus, the savings-to-output ratio of individuals with high ability converges to, or becomes larger than, that of those with low ability.

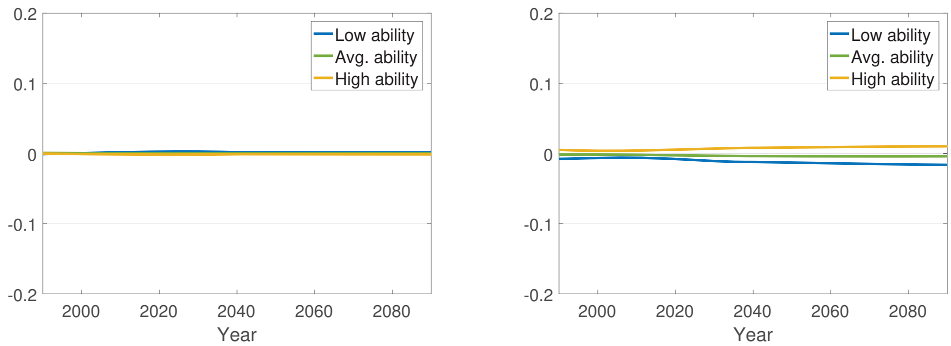
Figure 8 shows that the contribution of the Social Security pension system to the discrepancy between income and output for each ability group. Although Figure 8(b) shows that the flat replacement rate redistributes resources each year from individuals with low ability to individuals with high



(a) Baseline (b) Flat replacement

Figure 7: Factor I (savings-to-output): Period 1990-2090.

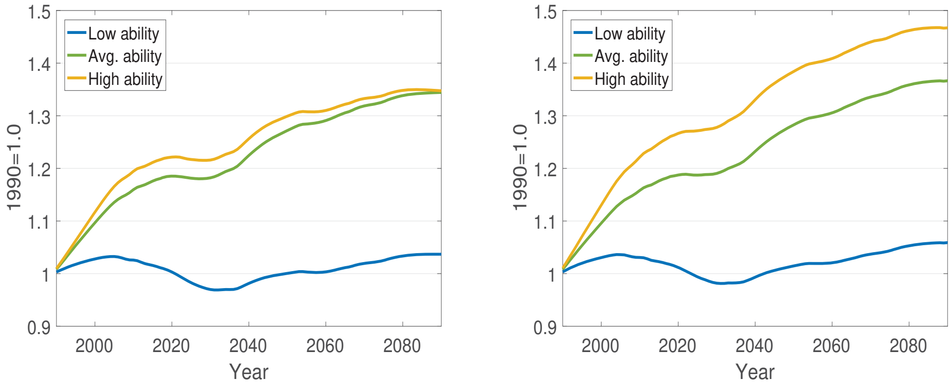
ability, the impact of the pension system on income per adult is small relative to the effect of the evolution of the savings-to-output ratio.



(a) Baseline (b) Flat replacement

Figure 8: Factor II (Social Security): Period 1990-2090.

Plugging the results plotted in figures 7-8 and the output per adult from Table 2 in Eq. (36), we obtain the growth of the income per adult by ability level. Figure 9 shows that the income per adult increases from 1990 to 2090 in both scenarios for all ability levels due to the evolution of the savings-to-output ratio for each ability group, as Figure 7 shows. Thus, the increase in the mean-age of the adult population, which is caused by the population aging process, will raise income inequality by ability level. The group of individuals with low ability are particularly affected by population aging since their income stays almost constant from year 1990 to 2090. Indeed, the income gap between the high ability group and the low ability group will increase by 30% in the baseline, but it would increase by 40% with a flat replacement rate. The greater income gap with a flat replacement rate stems from the fact that the flat replacement rate creates a higher incentive to accumulate human capital for the high ability group. Nevertheless, the implementation of a flat replacement rate leads to a faster growth in income for all ability groups.



(a) Baseline (b) Flat replacement

Figure 9: Income per adult (productivity de-trended): Period 1990-2090

7 Conclusions

This paper builds a computable overlapping generation model of labor supply with endogenous human capital formation in which individuals are heterogeneous by their learning ability level and life expectancy. The model is applied to the US and analyzes the economic consequences of running a pension system with a progressive replacement rate versus a flat replacement rate in the context of heterogeneous learning ability levels with an increasing life expectancy heterogeneity over time.

Our simulations suggest the following results. First, despite the fact that the US Social Security pension replacement rate is progressive, our paper shows that in the decades to come the Social Security will not redistribute from high skilled individuals to low skilled individuals. This is because high skilled individuals live, on average, longer after retirement than low skilled individuals

(Lleras-Muney, 2005). Therefore, high skilled individuals receive their pension benefits over a longer period of time, which compensates for their lower pension benefits relative to their lifetime income. Second, similar to Pestieau and Ponthiere (2012) and Ayuso et al. (2016), we find that by implementing a pension system with a flat replacement rate the pension benefits of high skilled individuals are subsidized by lower skilled individuals due to the increasing longevity inequality. This result holds true not only from a cross-sectional perspective but also from an individual point of view. Third, future income per capita will increase for all ability levels due to the further accumulation of human capital. However, simultaneously, the income inequality by ability level will increase further during the twenty first century because, as each ability group becomes older, the average financial wealth position will increase faster for those with higher abilities than for those with lower abilities due to their higher wage rates.

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A Agent's problem: Optimal allocation of resources

Agents start making decision when they complete the elementary school (8th grade) at the age of 15. They choose the consumption path (c), the additional education (e) they still want to acquire, and the leisure time (z). The remaining available time is devoted to work.

The expected utility V of an agent that belongs to group (θ, μ) at age x depends on the assets holding a, the pension earnings p, and the stock of human capital h. For simplicity in the exposition we get rid of the time variable and denote the next period with the symbol "$'$". Our agent solves the following problem:

For $15 \leq j \leq J_R$:

$$V(a,p,h) = \max_{c,z,e} \{U(c,z) + \beta\pi'(\mu)V'(a',p',h')\} \tag{A-1a}$$

subject to

$$a' = R(\mu)a + (1-\tau)R^H h(1-z-e) - c, \tag{A-1b}$$
$$p' = Ip + \rho R^H h(1-z-e), \tag{A-1c}$$
$$h' = h(1-\delta) + q(h,e;\theta), \tag{A-1d}$$
$$\lambda_1(1-z-e) = 0, \tag{A-1e}$$
$$\lambda_2 z = 0, \tag{A-1f}$$
$$\lambda_3 e = 0. \tag{A-1g}$$

For $j > J_R$:

$$V(a,p,h) = \max_{c} \{U(c,1) + \beta\pi'(\mu)V'(a',p',h')\} \tag{A-1h}$$

subject to

$$a' = R(\mu)a + b - c, \tag{A-1i}$$
$$p' = p, \tag{A-1j}$$
$$h' = h(1-\delta), \tag{A-1k}$$

and the boundary conditions

$$a_{15} = p_{15} = 0, h_{15} = 1, \text{ and } a_{\Omega+1}, p_{\Omega+1}, h_{\Omega+1} \geq 0, \tag{A-1l}$$

where β is the subjective discount factor, $\pi(\mu)$ is the conditional probability of surviving to the next period, $R(\mu)$ is the capitalization factor, τ is the social security contribution rate, R^H is the rental rate on human capital, δ is the human capital depreciation rate, and $q(h, e)$ is the human capital production function (with $q_h, q_e > 0$ and $q_{hh}, q_{ee}, q_{he} < 0$).

A.1 First-order conditions and envelope conditions

Let us denote Σ and Γ as the marginal rate of substitution between pension wealth and assets, V_p/V_a, and human capital and assets, V_h/V_a, respectively. The first-order conditions and envelope conditions are

$$U_c = \beta\pi'(\mu)V'_{a'}, \tag{A-2}$$

$$U_z = \begin{cases} U_c R^H h(1-\tilde{\tau}) + \lambda_1 - \lambda_2 & \text{for } 15 \leq j \leq J_R, \\ 0 & \text{for } j > J_R. \end{cases} \tag{A-3}$$

$$\Gamma' \frac{\partial h'}{\partial e} = \begin{cases} \frac{U_c R^H h(1-\tilde{\tau}) + \lambda_1 - \lambda_3}{U_c} & \text{for } 15 \leq j \leq J_R, \\ 0 & \text{for } j > J_R. \end{cases} \tag{A-4}$$

and

$$V_a = R\beta\pi'(\mu)V'_{a'}, \tag{A-5}$$

$$V_p = \begin{cases} I\beta\pi'(\mu)V'_{p'} & \text{for } 15 \leq j \leq J_R, \\ \beta\pi'(\mu)V'_{a'}\left(\frac{\partial b}{\partial p} + \Sigma'\right) & \text{for } j > J_R. \end{cases} \tag{A-6}$$

$$V_h = \begin{cases} \beta\pi'(\mu)V'_{a'}\left[R^H(1-z-e)(1-\tilde{\tau}) + \Gamma'\frac{\partial h'}{\partial h}\right] & \text{for } 15 \leq j \leq J_R, \\ \beta\pi'(\mu)V'_{h'}(1-\delta) & \text{for } j > J_R. \end{cases} \tag{A-7}$$

where $\tilde{\tau} = \tau - \rho\Sigma'$ is the effective social security tax on labor.

Assuming Σ and Γ are equal to zero in the very last period, combining the envelope conditions with the first-order conditions, taking into consideration the Kuhn-Tucker conditions and given $\lim_{z\uparrow 0} U_z = \infty$, the law of motion of the marginal rate of substitutions for the state variables are:

$$\Sigma = \begin{cases} \frac{I}{R(\mu)}\Sigma' & \text{for } 15 \leq j \leq J_R, \\ \frac{1}{R(\mu)}\left(\frac{\partial b}{\partial p} + \Sigma'\right) & \text{for } j > J_R, \end{cases} \tag{A-8}$$

$$\Gamma = \begin{cases} \frac{1}{R(\mu)}\left(\Gamma'(1-\delta) + \frac{U_z}{U_c}\frac{1-z}{h}\right) & \text{for } 15 \leq j \leq J_R, \\ 0 & \text{for } j > J_R. \end{cases} \tag{A-9}$$

Thus, by recursively calculating (A-8) and (A-9) backward we can calculate the rate at which our individual is willing to give up social security wealth or human capital in exchange of a marginal increase in her financial wealth, maintaining the same level of utility.

B Parametric components of the pension system

Given a retirement age J_R, the benefits claimed at retirement can be modeled with the following information: i) current contribution weights, ρ_j; ii) the capitalization index, I_t; iii) pensionable earnings, p; iv) the replacement rate, $\psi(p)$; and v) the retirement incentives or penalties. For the sake of simplicity and because we want to see the effect of different retirement lengths, we skip item (v).[9] Table 3 shows the parametrization of two of the main components of the US pension system (OASDI). Note that the pensionable earnings are derived by the accumulation of labor income histories, which are endogenously determined in the model. The value of $\rho_j = \rho$ for all j is chosen so as to obtain a pension cost to output ratio in year 2013 of 5%. This is done with a value of ρ of 1/35 or, equivalently, the average of 35 years of work. Not surprisingly, this value coincides with the US pension system that takes into account the 35 years of highest labor income along the working life of an individual. In our case, the highest labor incomes are close to retirement, hence we assume that $\rho = 0$ is initially zero and it is 1/35 during the last 35 years at work.

As Table 3 shows the maximum replacement rate is 0.90 when an individual's pensionable earnings is less than six times the average labor income, while the replacement rate tends to zero the larger the pensionable earnings

[9]Recent reforms have established a gradual increase in the normal retirement age that depends on the year of birth of the retiree. This information can be checked at www.socialsecurity.gov.

Table 3: Parametric components of the pension systems

Case	Replacement rate $\psi(p)$	Capitalization index I_t
Baseline (US)	$\begin{cases} 0.90 & \text{If } p \leq \frac{\bar{y}_t}{6}, \\ 0.32 + \frac{0.58}{6}\frac{\bar{y}_t}{p} & \text{If } \frac{\bar{y}_t}{6} < p < \bar{y}_t, \\ 0.15 + \frac{1.60}{6}\frac{\bar{y}_t}{p} & \text{If } \bar{y}_t < p < 2\bar{y}_t, \\ \frac{3.40}{6}\frac{\bar{y}_t}{p} & \text{If } 2\bar{y}_t < p. \end{cases}$	$\bar{y}_{t+1}/\bar{y}_t$
Flat replacement rate	0.417	$\bar{y}_{t+1}/\bar{y}_t$

are. Moreover, past pension earnings are updated each year to reflect the increasing cost of leaving through a capitalization index I_t. Let $\bar{y}_t$ be the average labor income of the economy in year t, which is calculated as

$$\bar{y}_t = \frac{\text{Total labor income in year } t}{\text{Total number of workers in year } t}.$$

Thus, the US pension system takes as capitalization index the annual increase in the average labor income; i.e. $I_t = \bar{y}_{t+1}/\bar{y}_t$.

Early retirement penalties and delayed retirement credits

B.1 Behavioral effects: Progressive vs. flat replacement rate

In this section we analyze the impact of the US pension benefit formula and the flat replacement rate formula on the decision making process.

The pension system changes the decision making process through the effective social security tax on labor,

$$\tilde{\tau} = \tau - \rho\Sigma', \tag{A-10}$$

which directly affects leisure, the educational investment, and hence the intensive labor supply (see eqs. (14)-(15)), while indirectly affects human capital stock, pension earnings, financial wealth, and consumption. According to Eq. (A-8), the pension benefit formula may have a differential effect on the decision making of each heterogeneous individual through the marginal rate of substitution between the social security wealth and the financial wealth, Σ.

There are two main factors that differentiate Σ across our heterogenous individuals. Since individuals purchase annuities, the first factor is the survival probability which depends on the frailty level. Thus, individuals with lower life expectancy will experience a faster increase in Σ than those with high life expectancy. The intuition is simple, the probability of reaching the retirement age and start claiming benefits increases faster with age for individuals with low life expectancy than for those with high life expectancy. The second factor is the derivative of the pension benefits with respect to the pension earnings,

$$\frac{\partial b}{\partial p} = \psi'(p)p + \psi(p). \tag{A-11}$$

Thus, depending on the pension replacement rate, Σ will differ across individuals based on the pension earnings that they have endogenously accumulated until retirement. Using the information from Table 3 in Eq. (A-11) we obtain that an increase of one unit in pension earnings leads to the following increases in benefits by replacement rate:

- Progressive replacement rate

$$\frac{\partial b}{\partial p} = \begin{cases} 0.90 & \text{if } p < \bar{y}/6, \\ 0.32 & \text{if } \bar{y}/6 < p < \bar{y}, \\ 0.15 & \text{if } \bar{y} < p < 2\bar{y}, \\ 0 & \text{if } p > 2\bar{y}, \end{cases} \tag{A-12}$$

- Flat replacement rate

$$\frac{\partial b}{\partial p} = 0.417. \tag{A-13}$$

From (A-8) and comparing (A-12) to (A-13) we can study the preference of each individual between the progressive replacement rate and the flat replacement rate. Specifically, ceteris paribus the survival probability, individuals who have accumulated pension earnings below one-sixth of the average labor income value the US pension system 2.16 (= 0.90/0.417) times more than the pension system with a flat replacement rate. For those individuals who have accumulated pension earnings between one-sixth and one average labor income and between one and two times the average labor income value the US pension system 0.77 (= 0.32/0.417) and 0.36 (= 0.15/0.417) times more, respectively, than a system with a flat replacement rate. As a result, these differences determine, together with the life expectancy, whether an individual values the social security as a tax or as a subsidy on labor. If $\tilde{\tau}$ is negative (i.e. subsidy), the pension system will provide individuals with an incentive for working more hours. In contrast, if $\tilde{\tau}$ is positive (i.e. tax), the pension system will reduce the effective wage rate and hence the working hours.

Figure 10 shows the effective social security tax/subsidy rate from age 30 to 65 for the two different pension benefit formulas and two selected birth cohorts. The panels on the left-hand side correspond to the baseline simulation, whereas the panels on the right-hand side correspond to the flat replacement rate. The effective social security tax/subsidy for the cohorts born in year 2000 are plotted in the top panels, while those born in year 2050 are plotted in the bottom panels. Several conclusions can be found looking at Figure 10. First, according to our simulations, the US pension system will reduce the effective wage rate per hour worked for all ability groups over the working life, which will lead to a fall in the intensive labor supply. Second, a flat replacement rate will reduce the effective wage rate per hour worked at the beginning of the working life but it will raise it at the end of the working life. As a consequence, workers will supply their labor more intensively at the end of their working life than without the pension system. Third, the US pension system taxes more heavily to individuals with high learning ability, while the flat replacement rate does so to individuals with low learning ability.

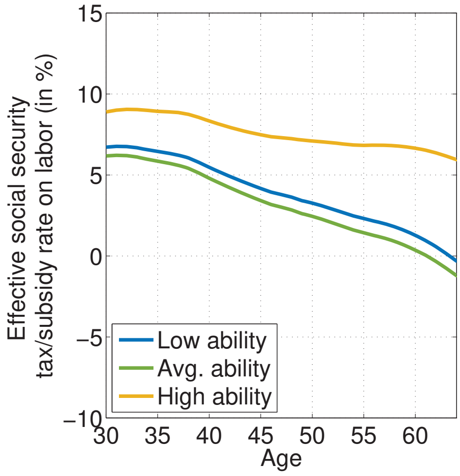
Frisch elasticities. Given an utility level, the extent to which a change in the effective social security tax/subsidy will modify our agent's decision variables is given by the Frisch elasticities. Thus, we next detail the Frisch elasticities that are derived in our model.

In the interior solution, Eq (29) yields the following Frisch elasticities on education and labor supply

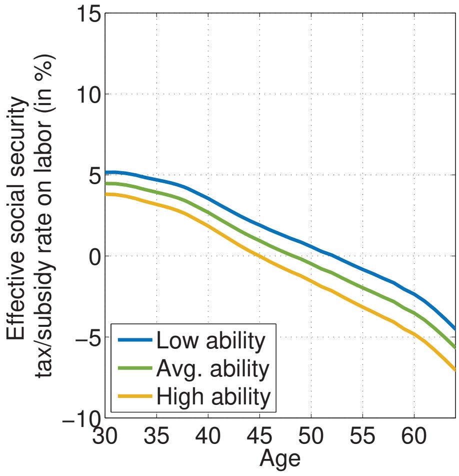
$$\frac{R_t^H(1-\tilde{\tau}_{t,j})}{e_{t,j}}\frac{\partial e_{t,j}}{\partial R_t^H(1-\tilde{\tau}_{t,j})} = -\frac{1}{1-\gamma}, \tag{A-14}$$

$$\frac{R_t^H(1-\tilde{\tau}_{t,j})}{\ell_{t,j}}\frac{\partial \ell_{t,j}}{\partial R_t^H(1-\tilde{\tau}_{t,j})} = \frac{1}{\sigma}\frac{z_{t,j}}{\ell_{t,j}} + \frac{1}{1-\gamma}\frac{e_{t,j}}{\ell_{t,j}}, \tag{A-15}$$

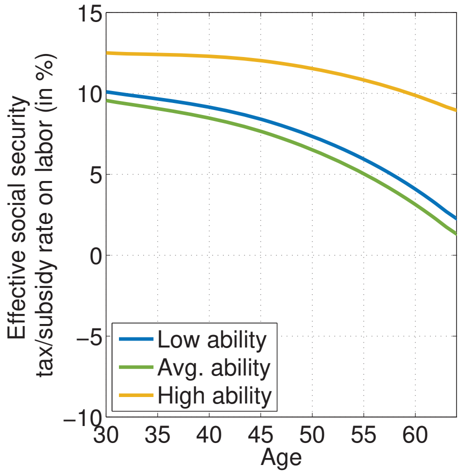
respectively. Given the parameters in Table 1, the intertemporal elasticity of substitution on consumption is one and the Frisch elasticity of education is -2.86. Thus, an increase of 1% in the (after-tax) wage rate per unit of human capital leads to a reduction in the educational investment close to 3% regardless of the age of the individual. In contrast, according to Eq (A-15) the Frisch elasticity of labor supply changes over the working life, across cohorts,



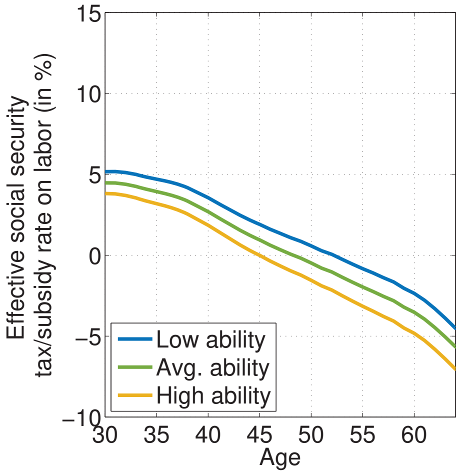
(a) Baseline, Cohort born in 2000



(b) Flat replacement, Cohort born in 2000



(c) Baseline, Cohort born in 2050



(d) Flat replacement, Cohort born in 2050

Figure 10: Effective social security tax/subsidy rate on intensive labor supply by pension system: Selected cohorts

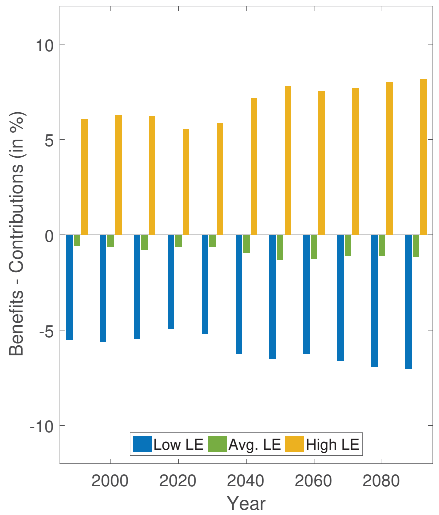
and by ability level. In particular, Table 4 shows that the model implicitly considers a Frisch elasticity of labor supply that starts at the entrance in the labor market well-above one and progressively falls until age 50. Moreover, this age tilt is more pronounced for younger cohorts and for individuals with higher learning abilities. As Keane and Rogerson (2012) and Eq. (A-15) show, in a model of labor supply with human capital investment, the value of the Frisch elasticity of labor supply is positively related to the time spent investing in human capital. This is because when the wage rate per human capital rises the educational investment is substituted with an increase in hours worked. As a consequence, since the optimal educational investment decreases with age, the Frisch elasticity of labor supply does as well. For the same reason, from Eq. (22), the age tilt of the Frisch elasticity becomes more pronounced because both a higher probability of surviving to retirement and a higher learning ability level rises the educational investment.

Table 4: Frisch elasticity of labor supply by age, cohort, and ability

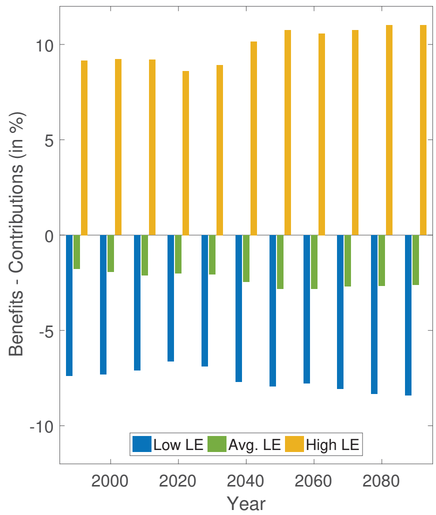
	Low ability			Medium ability			High ability		
Age	30	40	50	30	40	50	30	40	50
Cohort									
1950	1.30	0.89	0.80	1.95	1.00	0.71	2.41	1.06	0.68
1975	1.37	0.88	0.76	1.96	0.97	0.69	2.37	1.04	0.67
2000	1.79	0.96	0.69	2.45	1.06	0.64	3.00	1.13	0.64
2025	1.99	0.98	0.65	2.62	1.07	0.61	3.31	1.16	0.62
2050	2.19	1.01	0.62	2.88	1.10	0.60	3.68	1.20	0.61
2075	2.30	1.01	0.61	3.09	1.11	0.59	4.10	1.22	0.60

C Main figures by life expectancy

Figures (11)-(14) are not shown in the main text because it is well-known that pension systems, while reduce inequality between the surviving old, tend to increase inequality between those who survive to retirement and those who died prematurely (Pestieau and Ponthiere, 2016). Nevertheless, given that individuals with high learning ability are benefitted in a system with a flat replacement rate compared to the baseline, it can be observed that inequality is widened in the flat replacement scenario between the low and high life expectancy groups.

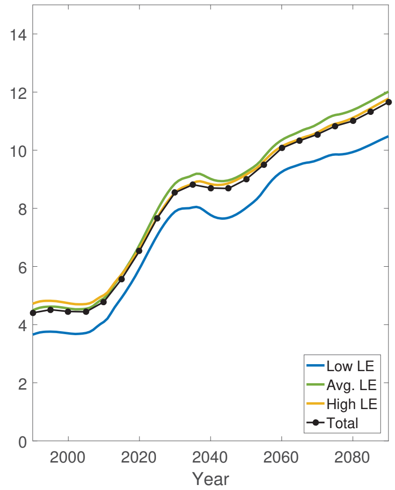


(a) Benefits minus contributions, baseline

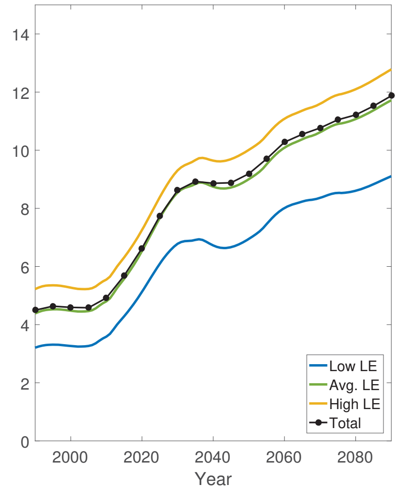


(b) Benefits minus contributions, flat replacement

Figure 11: Benefits minus contributions by ability group (in percentage of the total pension budget): Calendar years 1990-2090

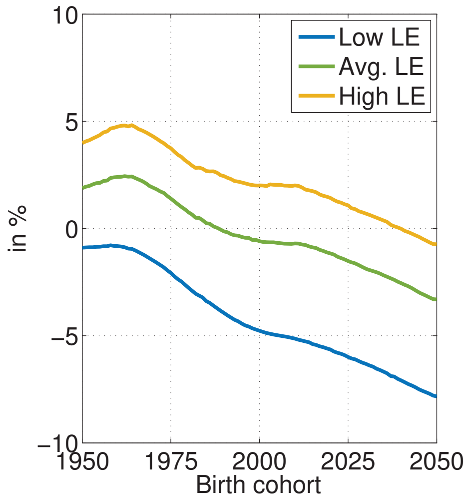


(a) Total pensions to output ratio, baseline

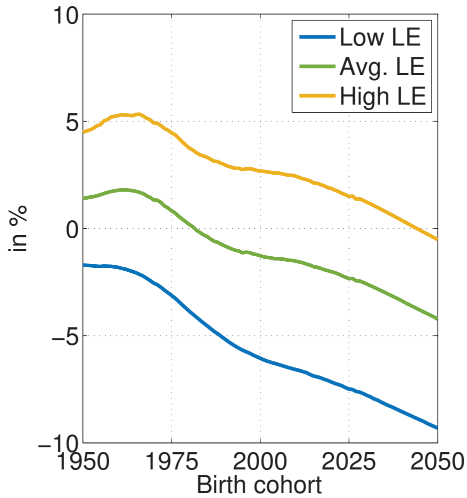


(b) Total pensions to output ratio, flat replacement

Figure 12: Total pension to output ratio from 1990 to 2090 (in %)

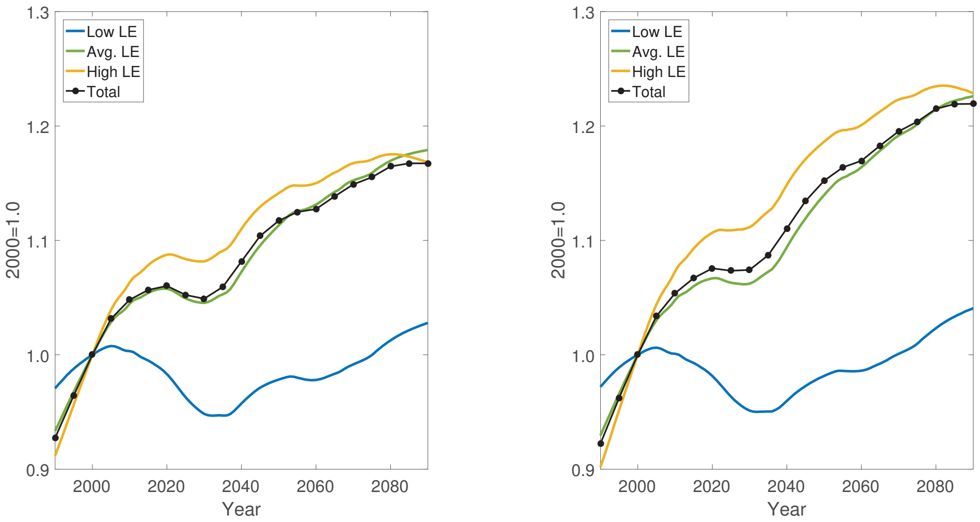


(a) SSW to P.V. labor income ratio, baseline



(b) SSW to P.V. labor income ratio, flat replacement

Figure 13: Social security wealth as a fraction of present value of labor income (in %): 1950-2050 birth cohorts



(a) Per capita income, baseline

(b) Per capita income, flat replacement

Figure 14: Income per capita (productivity de-trended): Period 1990-2090

D Proof: Income per adult

An important result in a model with heterogenous population groups is that the output produced by each group does not necessarily coincide with the income generated by each group. There are two sources of discrepancy: the savings-to-output ratio and the net transfers position.

Proof. To see this important result it is convenient to generate the national accounting. First, let us use the household budget constraint and multiply it by the population at the end of the interval

$$a_{t+1,j+1}N_{t+1,j+1}(\theta) \\ = \begin{cases} \frac{1+r_t}{\pi_{t,j}(\mu)}a_{t,j}N_{t+1,j+1}(\theta) + (1-\tau_t)R_t^H h_{t,j}\ell_{t,j}N_{t+1,j+1}(\theta) - c_{t,j}N_{t+1,j+1}(\theta) & \text{for } j \leq J_R, \\ \frac{1+r_t}{\pi_{t,j}(\mu)}a_{t,j}N_{t+1,j+1}(\theta) + b_{t,j}N_{t+1,j+1}(\theta) - c_{t,j}N_{t+1,j+1}(\theta) & \text{for } j > J_R. \end{cases} \tag{A-16}$$

Summing across age groups and taking into consideration that $K_t(\theta) = \sum_j a_{t,j}N_{t,j}(\theta)$ and $C_t(\theta) = \sum_j c_{t,j}N_{t+1,j+1}(\theta)$, we have

$$K_{t+1}(\theta) = (1+r_t)K_t(\theta) + R_t^H h_t \ell_t N_{t+1}(\theta) + SS_t(\theta) - C_t(\theta) \tag{A-17}$$

where the net transfer position T of the population group (θ) is given by

$$SS_t(\theta) = \sum_{j>J_R} b_{t,j}N_{t+1,j+1}(\theta) - \tau_t R_t^H \sum_{j\leq J_R} h_{t,j}\ell_{t,j}N_{t+1,j+1}(\theta) \tag{A-18}$$

Using (A-17) we can express the budget constraint in terms of expenditures (left-hand side) and incomes (right-hand side) as follows

$$C_t(\theta) + I_t(\theta) = (r_t + \delta)K_t(\theta) + R_t^H h_t \ell_t N_{t+1}(\theta) + SS_t(\theta) \tag{A-19}$$

where $I_t(\theta)$ denotes savings, i.e. $K_{t+1}(\theta) - (1-\delta)K_t(\theta)$.

Under the assumption that all workers are perfectly substitutable, regardless their learning ability level, the output produced in year t by the population group (θ) is given by

$$Y_t(\theta) = \frac{R_t^H}{1-\alpha} h_t \ell_t N_{t+1}(\theta). \tag{A-20}$$

Then, dividing the expenditures and incomes generated by the group (θ), or Eq. (A-19), by its output produced, or Eq. (A-20), gives, after using (6) and rearranging terms,

$$\frac{C_t(\theta) + I_t(\theta)}{Y_t(\theta)} = 1 + \underbrace{\alpha\left(\frac{K_t(\theta)/Y_t(\theta)}{K_t/Y_t} - 1\right) + \frac{SS_t(\theta)}{Y_t(\theta)}}_{\text{Source of discrepancy}}. \quad \text{(A-21)}$$

Therefore, the population group (θ) will consume (receive more income) more than will produce when their savings-to-output ratio is higher than the average capital-to-output ratio and when they are net transfer receivers to the pension system.

■

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