Financial and Nuclear Meltdowns: 
the fragility of chain-reaction critical processes

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Summary

This paper explores the dynamic roots of two recent catastrophic events: the financial meltdown triggered by the subprime mortgage crisis and the partial nuclear meltdown of three reactors of the Fukushima1 plant. We suggest that what makes fragile, and thus accident-prone, both nuclear reactors and financial systems is the criticality of their chain-reaction dynamics. We claim that a systematic examination of the dynamic analogies between the nuclear and financial chain reactions has an heuristic potential that has been unduly neglected. In particular the common features of their dynamic behaviour impose similar constraints on their controllability and calls for a more precautionary policy in their design and regulation.
The financial “tsunami” that hit the USA and Europe in 2008 and culminated in the bankruptcy of Lehman Brothers in September 2008, triggered a “meltdown” of the financial system that was somehow thwarted only by an unprecedented public bail-out of many big financial institutions. In March 2011, while the effects of the financial crisis were not yet fully re-absorbed, a real tsunami hit the County of Sendai in North-East Japan and triggered the partial meltdown of the nuclear reactors 1, 2, and 3 of the Fukushima1 plant. In our opinion, the analogies between these two episodes go much beyond terminology. Although the common features of nuclear and financial chain reactions have been almost completely neglected in the scientific literature we claim that we may draw from them pregnant insights. In what follows we focus on their foundations and hint at some of their implications.

We want to show in particular that, in order to understand and prevent catastrophic events in finance and nuclear energy generation, we have to focus on the critical chain reactions characterizing accident-prone systems. This sort of structural instability has to do both with the complex links between the parts of the system (as emphasized, among others, by Haldane & May1, Johnson2 and Lux3) and with the complex dynamics of the system as a whole. A thorough analysis of critical dynamics should combine both aspects but a full-fledged implementation of this promising research strategy needs more time than we have at our disposal to avoid further catastrophes in the near future (Sornette & von der Becke4). We claim in this paper that the analysis of the dynamic properties of critical chain reactions in fragile systems may give important insights on their dynamics and controllability. This may be shown through elementary models that avoid any confusion between complex and complicated dynamics. These models show that the critical processes characterizing nuclear reactors and finance dynamics are critical or structurally unstable in the sense that an infinitesimal shock perturbing a critical process is sufficient to change radically the dynamic behaviour of the system (a critical survey of different notions of instability and their implications may be found in Vercelli5). The stabilization strategy of these processes has proved so far unable to prevent a multitude of minor crises and the emergence of rarer deep crises leading in both cases to a “meltdown”. This calls for a much more effective preventive strategy.

The chain reaction of a nuclear reactor

A nuclear meltdown is an informal term for a severe accident bringing about a, generally partial, melting of the nuclear reactor’s core seriously jeopardizing the process of energy generation and its safety. The term is not officially defined by the Nuclear Agencies, such as the International Atomic Energy Agency (IAEA) or the U.S. Nuclear Regulatory Commission, but is commonly used by journalists and experts. In order to understand under which circumstances a nuclear meltdown may
Both nuclear energy generation and nuclear weapons exploit the properties of nuclear fission. The nuclear fission is rooted in the high “fragility” of the nuclides (or isotopes) of heavy elements such as Uranium ($^{235}\text{U}$) and Plutonium ($^{239}\text{Pu}$). When a heavy nuclide is hit by a neutron it is likely to undergo a process of “fission” that breaks the nucleus into two or more fragments, emits free neutrons and releases at the same time a great quantity of energy in the form of radiation (gamma rays and neutrinos) and heat. The most important fission reaction for nuclear energy generation are those of uranium-235, when hit by a slow-moving (thermal) neutron:

\[
^{235}\text{U} + \text{neutron} \rightarrow \text{fission fragments} + 2.4 \text{ neutrons} + 192.9 \text{ MeV}.
\]

This reaction releases a huge amount of energy (hundreds of millions of eV, i.e. electronvolts, while chemical reactions release an amount of energy not exceeding a few eVs). In addition, if the ejected neutrons hit nearby heavy nuclides they produce with a high degree of probability one or more further nuclear fissions. This may trigger a chain reaction that under given conditions may be self-sustained. A nuclear chain reaction is thus a formidable source of energy that may be used for civil purposes. The trouble is that it also releases a great amount of radiation as the fission fragments are subject to radioactive decay while much of the energy released has the form of radiation (gamma rays and neutrinos). The difficult challenge of nuclear engineering is that of producing a great amount of energy in a continuous way without releasing radiation outside the nuclear plants. This is by no means an easy task since the physics of nuclear plants shows how intrinsically unstable is the dynamic process of nuclear energy generation.

The crucial part of a nuclear reactor is its core consisting of an assembly of fuel rods. The core is usually surrounded by a neutron moderator (regular water, heavy water, graphite, and so on) that reduces the kinetic energy of newly produced neutrons in consequence of fission events since slower neutrons are more likely to induce further fissions. In addition a nuclear reactor is typically characterized by an exogenous source of neutrons: a primary source that speeds up the start of a critical chain reaction or a secondary source that improves the convergence towards the critical state and its sustainability through time.

In order to study the chain reaction of a nuclear reactor the analysis has to focus on the population of free neutrons $N$. The dynamic behaviour of a nuclear reactor may be described in the simplest possible way by the following differential equation:\(^6,^7\):

\[
dN/dt = \alpha N/\tau + N' \tag{1}
\]

where $N$ is the number of free endogenous neutrons in a reactor core, $N'$ is the number of neutrons injected in the core by an external source, $\tau$ stands for the average lifetime of each neutron before it
escapes from the core of the reactor or is absorbed by a nucleus, while the parameter \( \alpha \) is a constant of proportionality. In order to allow a more intuitive understanding of the complex dynamics of a nuclear reactor we translate this differential equation in a difference equation assuming that the length of the period is \( \tau \) (so that \( \tau = 1 \)). We get:

\[
N_t = kN_{t-1} + N' \quad (2)
\]

where the parameters \( N' \) and \( k \) are assumed to be constant. In nuclear engineering the parameter \( k \) is called “effective multiplication factor” and expresses the average number of neutrons released by one fission that bring about another fission. This number is crucial to study the dynamic properties of the core. When \( k < 1 \), the system is subcritical and cannot sustain a chain reaction. In this case the system is stable but the energy released rapidly fades away. The number \( F \) of fission events triggered by an exogenous neutron is given by \( F = N'/(1-k) \), where \( 1/(1-k) \) may be defined as the "multiplier" of exogenous neutrons that determines the equilibrium population of neutrons within the reactor. When \( k > 1 \) the system is supercritical and triggers a chain reaction that increases exponentially the number of fissions and thus also the population of neutrons progressively amplifying the energy released and undermining its control. The chain reaction may be exploited for a sustainable production of energy only in the borderline case, when \( k = 1 \). In this case the system is critical and the mean number of free neutrons remains constant bringing about, ceteris paribus, a stationary process of fission events and energy release. The only useful state of the core of a nuclear reactor is thus a bifurcation point that nuclear engineering tries hard to stabilize.

The dynamic behaviour of the reactor’s core under the three different hypotheses mentioned above may be represented in a simplified way as in fig.1.

Fig 1 about here

We measure on the ordinates axis \( N_{t+1} \) and on the abscissa axis \( N_t \). The equation \((2)\) has a slope that depends on \( k \), while the locus of possible equilibrium values (stationary since we have assumed that the exogenous neutron generation rate \( N' \) is constant) is represented by the bisecting line where \( N_{t+1} = N_t \). The subcritical case represented in fig 1a is characterized by a stable equilibrium \( N* \) that is a function of the rate of exogenous generation of neutrons

\[
N* = N'/(1-k) \quad (3)
\]

The supercritical case represented in fig.1c has no realizable equilibrium while the population of free neutrons and the number of fission events grows exponentially. In the critical case represented in 1b equilibrium is inexistent when \( N' > 0 \) or indeterminate when \( N' = 0 \). The critical case is a
borderline singularity that is structurally unstable as an infinitesimal perturbation to $k$ may transform the system in supercritical or subcritical\(^5\).

The fine tuning of $k$ is very difficult since the physical processes underlying the aggregate value of $k$ are probabilistic and are subject to complex dynamics. The parameter $k$ depends on the following main factors\(^6,7\):

$$k = P_i P_f \eta - P_a - P_e$$  \hspace{1cm} (4)

where $P_i$ is the probability that a particular neutron strikes a fuel nucleus, $P_f$ is the probability that the struck nucleus undergoes a fission, $\eta$ is the average number of neutrons ejected from a fission event (it is between 2 and 3 for the typical fuel utilized in nuclear plants: $^{235}$U and $^{239}$Pu); $P_a$ is the probability of absorption by a nucleus of the reactor not belonging to the fuel, and $P_e$ is the probability of escape from the reactor’s core. In other words, the product of the first three variables measures the strength of the fission chain reaction, while the probability of absorption and escape measure the average leakage from the system. In consequence of the probabilistic nature of its underlying process, $k$ necessarily fluctuates off its critical value. When $k<1$ the efficiency in energy generation declines, when $k>1$ the safety of the reactor is undermined. A nuclear reactor thus requires reliable mechanisms of regulation that keep the average of the fluctuations of $k$ at its critical value while constraining as much as possible their amplitude.

**The propagation process in a monetary economy**

A financial “meltdown” is an informal term used in finance to designate a severe crisis that undermines the capability of the financial system to support the real economy triggering a serious recession. This term is not rigorously defined in academic economics but is in common usage among practitioners, experts and journalists. This terminology has been probably imported from nuclear physics to emphasize a situation, similar to that of a nuclear meltdown, in which the financial system becomes unable to play its crucial role of support to the real economy while the decision makers lose control of its dynamics. In this case, however, the metaphor should not be taken literally since a financial meltdown is typically characterized by a credit crunch and a sudden loss of liquidity: it is a freeze rather than a meltdown. In order to understand under which conditions a financial meltdown may happen we have to focus on the circuit of economic and financial transactions.

The economic activity is characterized by a mechanism of propagation of impulses that has several analogies with the nuclear chain reaction discussed above. While in a nuclear reactor the process of propagation of an impulse is based on the alternation between fission events of nuclides hit by free neutrons and the consequent ejections of free neutrons originating new fission events, in
The process of propagation of an impulse is based on the alternation of income flows $y$ received by economic units and expenditure flows $e$ financed by the previous income flows. Focusing on the real side of the economy, the cumulative effects of this alternation triggered by an impulse $e'$ representing the exogenous expenditure (autonomous investment plus public expenditure) converge towards a finite measure $y^*$ when the marginal propensity to consume $c < 1$: 

$$y^* = e'/(1 - c), \quad 0 < c < 1,$$

where $1/(1 - c)$ is the so-called “multiplier” introduced by Kahn\(^8\) and Keynes\(^9\) to study the effects of public expenditure and to determine aggregate income. Here $c$ expresses the propensity of economic units to translate the inflows of income in outflows of expenditure and plays the same dynamic role of the effective multiplication factor $k$ in the equations 2 and 3 describing the dynamic behaviour of a nuclear reactor. In this simple version of the multiplier model the stability of the real system is assured by a positive marginal saving rate implying a net leakage from the system. The analogy with the subcritical case of a nuclear chain reaction is striking as in both cases the propagation process has a similar dynamic structure (see fig.2).

A positive aggregate saving rate is the normal case observed in the past most of the time in most countries. However, in the last decades the saving rate greatly diminished in developed countries progressively pushing the real economic system towards a critical regime, so reducing its stability. In a few countries, and most notably in the USA, the saving rate became slightly negative, or almost so\(^10\), just before the outbreak of the subprime crisis in 2007 contributing to the subsequent economic and financial instability. In addition we have to emphasize that the stabilizing role played by a positive saving rate crucially depends on the simplifying assumptions underlying the standard Kahn-Keynes multiplier model that all the investment is exogenous. This assumption restricts the validity of the model to the short period as the effects of income variation on the capital stock are neglected. The latter relation is usually expressed by “the acceleration principle” or “accelerator”.

Its simplest version is the following:

$$I_t = v(Y_t - Y_{t-1})$$

where $I_t$ stands for the induced investment and $v$ is the capital/output ratio. As soon as we consider the impact of endogenous investment on the income-expenditure chain reaction, the potential instability becomes evident, as has been first pointed out by Harrod\(^11\). When $I_t = S_t$, the aggregate endogenous expenditure $E_t$ is equal to the aggregate income in the previous period $Y_{t-1}$ and the system operates under a critical regime:
\[ v(Y_t - Y_{t-1}) = sY_t \]

from which we derive immediately:

\[ \frac{(Y_t - Y_{t-1})}{Y_t} = g = \frac{s}{v}. \]

where \( g = \frac{s}{v} \) is what Harrod called “warranted”, that is sustainable, rate of growth (fig.2b).

Unfortunately this steady state is a “razor’s edge”: an increase of expenditure over income, however small, would render the system supercritical determining an unsustainable rate of growth (fig.2c), while any reduction of expenditure would transform the system in subcritical (fig.2a). We do not pursue further this line of investigation from the point of view of the real economy because the instability of the economy crucially depends on the financial side of the income-expenditure process. In a modern monetary economy, an excess of endogenous investment over saving in a given period is made possible by the credit system. In logical terms an excess of expenditure over income could be financed by dishoarding reserves accumulated in the past. However, hoarding and dishoarding had a crucial role in the ancient world, while accumulation and depletion of reserves have only a secondary role in modern capitalism. A persisting excess of investment over saving or, more in general, of expenditure over income has to be financed through borrowing. To understand the intrinsic criticality of contemporary financialized economies we have thus to focus on the monetary and financial side of transactions and economic decisions.\(^{12,13,14,15}\)

The first monetary chain reaction that has been systematically explored in the economics literature is rooted in the alternation between credit and bank deposits. Additional credit translates in additional bank deposits that allow the concession of further credit and so on. According to the monetarists this process explains the money supply \( M \) as exogenously determined by the monetary base \( B \) assumed to be under the strict control of monetary authorities. The alternation mentioned above is characterized by a crucial leakage imposed by the legal reserve ratio \( \alpha \) of banks while other two significant leakages are the excess reserve ratio \( \beta \) and the currency drain ratio \( \gamma \). The credit multiplier may be thus expressed in the following way: \(^{16,17}\)

\[ M = B(1 + \gamma)/(\alpha + \beta + \gamma). \]

The system is subcritical since there is a leakage in the system represented by \((\alpha + \beta + \gamma)\) and the multiplication factor \(1-(\alpha + \beta + \gamma)<1\); however, the lower the desired reserves \( \alpha + \beta \) the more the system approaches a critical state. This is what happened in the recent years as financial innovation helped the financial institutions to elude the legal requirement while the excess reserves ratio tended to vanish and the currency drain ratio became increasingly irrelevant. In the USA and other countries this tendency contributed to increase the instability of the system. The nexus between the credit multiplier and financial crises has been hinted at since long but never seriously analyzed. For example Friedman and Schwartz observed that “a liquidity crisis in a unit fractional reserve banking
system is precisely the kind of event that triggers- and often has triggered- a chain reaction. And economic collapse often has the character of a cumulative process. Let it go beyond a certain point, and it will tend for a time to gain strength from its own development as its effects spread and return to intensify the process of collapse” (Friedman and Schwartz18). In a fractional-reserve banking system, in the event of a bank run, the demand depositors and note holders would attempt to withdraw more money than the bank has in reserves, causing the bank to suffer a liquidity crisis and, ultimately, to perhaps default.

The monetarist belief in the exogeneity of the monetary base fell in disrepute since the early 1980s. This assumption requires demanding conditions such as constant velocity of money circulation or at least its independence of the business cycle, while the empirical evidence suggests that it is quite volatile and strongly pro-cyclical. Goodhart19, e.g., wrote that the base money multiplier model is 'such an incomplete way of describing the process of the determination of the stock of money that it amounts to misinstruction'. The credit multiplier has been rejected in particular by the advocates of an endogenous money theory advanced since long and subscribed among others by Schumpeter and many post-Keynesians (for a recent assessment see Lavoie20).

Endogenous money theory states that the supply of money is credit-driven and determined endogenously by the demand for bank loans, rather than exogenously by monetary authorities. In this case the analogy with nuclear reactor’s instability is even stronger. The trouble with criticality is that, even in the absence of significant external shocks, a small change from within the system may be sufficient to trigger an unstable chain reaction. That is why criticality characterizes many catastrophe-generating systems21.

In a given period $t$, each economic unit is characterized by a financial inflow $y_t$ and a financial outflow $e_t$. The ratio $e_t/y_t$ is a significant index of its current financial condition as it affects both its liquidity and solvency15. It is also an index of the financial multiplication factor. Its value may be easily higher than unity and may persist in such a state for a relatively long time. In this case the dynamics of the financial system is supercritical, a “bubble” in the economic jargon that typically occurs during a boom. This is made possible by credit that creates inflows ex nihilo in the expectation that the consequent increase in outflows will generate in the future higher inflows that will permit the repayment of debt with an interest. The increase in the extant credit of the private sector typically happens in the period of vigorous economic expansion when the euphoria of the agents leads them to seek a higher leverage. As soon as the ensuing financial bubble(s) burst(s) the system becomes subcritical to reduce the excessive leverage. Also in this case, as in a nuclear reactor, the critical state is the only one sustainable in the long run, while a deviation from it tends to increase. In order to understand the sudden switch from a supercritical dynamics to subcritical
dynamics and vice versa, we have to introduce a second source of criticality that interacts with the first one. The current values of the liquidity ratio affects its expected values the sum of which determines the solvency of the economic unit. Whenever the solvency ratio $k^*$ that measures the ratio of discounted expected outflows and inflows is $<1$, the unit has a positive net worth and is solvent; $k^*$=1 is the critical value beyond which the unit becomes virtually insolvent since its net worth is negative. To avoid bankruptcy, the economic units have a desired value of the insolvency ratio sufficiently far from the critical value to withstand unexpected contingencies. The interaction between $k$ and $k^*$ determines the cyclical behaviour of financial conditions. This dynamic mechanism produces semi periodic minor financial crises during business cycles downturns and a few, much rarer, major financial crises that degenerate into recession or depression in consequence of contagion. To understand why we have to add to the first chain reaction induced by the expectations, a second chain reaction that depends on the financial linkages between units. In minor crises the contagion is limited in extent, time and space, while in the major crises its effects are pervasive and quite difficult to stop.

**Nuclear and economic chain reactions: analogies and implications**

The chain-reaction criticality characterizing the dynamics of both a nuclear reactor and a monetary economy raises similar issues of regulation and risk management. First, criticality implies that predictability and controllability is severely limited and active regulation is arduous and unreliable. In nuclear reactors the principal instrument of regulation is given by control rods that may be inserted to variable degree in the core of the reactor to slowdown the chain reaction as soon as it becomes supercritical or to accelerate it as soon as it becomes subcritical. In the economy the chain reaction may be slowed down and moderated by reducing the leverage of economic units and improving their solvency indexes. However, while successful regulation is manageable in both cases under routine circumstances, it may become prohibitive under unexpected scenarios. The regular working of the reactor is constantly monitored by highly trained technicians. They may, for example, insert control rods to reduce or increase $k$. Unfortunately these active interventions of regulation are subject to errors that can trigger an uncontrollable process leading to the partial meltdown of fuel. Serious mistakes have been made quite often even by the best trained technicians being unable to forecast the complex dynamics of a nuclear reactor following an unexpected event (that does not need to be a large shock). That is why the training of nuclear plants technicians includes an extensive programme of simulations to refine their ability to cope with unforeseen circumstances. It is impossible, however, to simulate all the possible scenarios and the risk of inadequate behaviour remains extremely high. The Chernobyl accident, for example, has been
triggered by an incautious stoppage of the reactor 2 to perform a test meant, ironically, to improve its safety.

Analogously, the subprime crisis has been triggered by systematic and reiterated misbehaviour of many subjects, including the over-exposition of financial institutions and households, the illusion that structured securities could spread risk in a more efficient way, the lax supervision of monetary authorities reluctant to interfere with private decisions. The intrinsic weakness of active regulation in these two fields has led the experts to focus on mechanisms of passive regulation that are automatically switched on in case of necessity.

In nuclear reactors the principal mechanism of passive regulation is provided by the neutron moderator (often regular water) surrounding the fuel bars. The controllability and safety of nuclear energy generation depends crucially on the amount and nature of the neutron moderator. There is an optimal amount of a given kind of moderator as less moderation reduces the probability of fission while more increases the probability of escape. In addition most moderators become less effective with increasing temperature so that if the reactor overheats the chain reaction tends to slow down: for example regular water, that is used as moderator in most reactors, starts to boil and reduces the effective multiplication factor. However, there may be an unexpected leakage of water or steam as well as a failure of the systems to pump new water into the reactor’s core as in the case of Fukushima1 after the flooding of the emergency pumps.

In the economic system passive regulation is delegated to the invisible hand of the market. However, only in the case of an ideal model of perfect-competition market we may rely on its virtues of self-regulation. Unfortunately real markets do not comply with the long list of demanding assumptions that define a perfect-competition market so that the “invisible hand” is often weak, trembling, and coerced by big companies or public agencies. In real markets, as in existing reactors, the failure of self-regulation may originate a cascade of further failures that may bring about their “meltdown”. In both cases the likelihood of local failures that may have much wider, even global, consequences calls for a global regulator that imposes strict standards to local units and has the power to enforce them. This is not the case neither in the nuclear energy field nor in economics and finance. In both cases the authorities are national and the efficiency of their interventions is jeopardized by local interests and regulatory capture.

**Concluding remarks**

In this paper we have focused on the far-reaching analogy between the nuclear and financial chain-reaction criticality claiming that reflections on this analogy and its implications may be suggestive from different points of view. In particular, it may provide a more concrete intuitive perception of
the causes and consequences of both nuclear accidents and financial crises. In the case of nuclear energy generation most people believe that the understanding of its risks is restricted to nuclear physicists. The analogy with finance, of which anyone has some direct experience, may help the layman to understand, if not its details, at least the nature of the risks involved in nuclear energy generation. In finance, on the other hand, the causal links are mediated by long chains of effects transmitted through layers of derivatives and networks of units; they depend, in addition, on expectations quite vulnerable to cognitive and emotional shocks. The analogy with nuclear reactor dynamics may help us to grasp the concrete implications of complex dynamics in financial processes. In particular, the consequences of a nuclear accident, at least their short-term effects, are much more tangible as the explosions and leakages of radioactive vapour are immediately visible while the leakages of radiation can be easily measured in an objective way, and their effects on human health (in particular cancer) are vividly perceived by everyone.

We believe that the analysis started in this paper may be significant to assess the risks involved with the operation of fragile accident-prone dynamic systems such as nuclear reactors or financial systems. The risks involved with nuclear energy generation are not just a matter of faulty design of existing nuclear reactors but they are intrinsic in the complex dynamics of its underlying process even when active and passive regulation seem to be carefully designed. The risks involved with sophisticated financial systems spring not only from the fraudulent or myopic behaviour of “rotten apples” but from the in-built criticality of financial processes. We have to understand that the frequent occurrence of nuclear accidents and financial crises are both deeply rooted in the structural instability of their underlying processes, and that a correct management of the hard risks involved by their complex dynamics requires the adoption of precautionary policies much stricter than the current ones 23.
References

10 Guidolin, M, & E.A. La Jeunesse The decline in the U.S. personal saving rate: is it real and is it a puzzle? Federal Reserve Bank of St.Louis Review, 89, 491-514.
a) subcritical case: \( k < 1 \)

b) critical case: \( k = 1 \)

c) supercritical case: \( k > 1 \)

Fig. 1  Dynamic regimes of a nuclear reactor

a) subcritical case: \( c < 1 \)  
(Kahn-Keynes multiplier) 

b) critical case: \( g = s/v \)  
(warranted rate of growth) 

c) supercritical case: \( g > s/v \)  
(upward knife edge) 

Fig. 2  Dynamic regimes of a monetary economy